FIRST PRACTICE EXAM FIRST HOURLY **Math 21a,Spring 2003**

Name:

MWF10 Ken Chung
MWF10 Weiyang Qiu
MWF11 Oliver Knill
TTh10 Mark Lucianovic
TTh11.5 Ciprian Manolescu

- Start by printing your name in the above box and check your section in the box to the left.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.
- The hourly exam itself will have space for work on each page. This space is excluded here in order to save printing resources.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		100

Problem 1) TF questions (20 points)

Circle for each of the 20 questions the correct letter. No justifications are needed. Your score will be $C - W$ where C is the number of correct answers and W is the number of wrong answers.

T	F	The vectors $\langle 3,-2,1\rangle$ and $\langle -6,4,2\rangle$ are parallel.
T	F	The length of the vector $\langle 3,4,0\rangle$ is 25.
T	F	For any two vectors, $\vec{v}\times\vec{w} = \vec{w}\times\vec{v}$.
T	F	The vectors $\langle 1,1\rangle$ and $\langle 1,-1\rangle$ are orthogonal.
T	F	For any two vectors $\vec{v},\vec{w}$ one has $\vert\vec{v}+\vec{w}\vert^2 = \vert\vec{v}\vert^2 + \vert\vec{w}\vert^2$.
T	F	The surface $x^2 - y^2 + z^2 = 1$ is a one-sheeted hyperboloid.
T	F	The set of points which have distance 1 from a line is a cylinder.
T	F	If $\vert\vec{v}\times\vec{w}\vert = 0$ for all vectors $\vec{w}$, then $\vec{v} = 0$.
T	F	Any nonempty intersection of two planes is always a line.
T	F	If $\vec{u}$ and $\vec{v}$ are orthogonal vectors, then $(\vec{u}\times\vec{v})\times\vec{u}$ is parallel to $\vec{v}$.
T	F	Two nonparallel lines in three dimensional space always intersect in a point.
T	F	Every vector contained in the line $\vec{r}(t) = \langle 1+2t, 1+3t, 1+4t\rangle$ is parallel to the vector $(1,1,1)$.
T	F	If in spherical coordinates a point is given by $(\rho,\theta,\phi) = (2,\pi/2,\pi/2)$, then its rectangular coordinates are $(x,y,z) = (0,2,0)$.
T	F	If the velocity vector $\vec{r}\,'(t)$ of the planar curve $\vec{r}(t)$ is orthogonal to the vector $\vec{r}(t)$ for all times t, then the curve is a circle.
T	F	Every point on the sphere of radius ρ is determined alone by its angle ϕ from the z axis.
T	F	The equation $r = 3$ in cylindrical coordinates is a sphere.
T	F	The set of points which satisfy $x^2 - 2y^2 - 3z^2 = 0$ form an ellipsoid.
T	F	A surface which is given as $r = 2 + \sin(z)$ in cylindrical coordinates stays the same when we rotate it around the z axis.
T	F	If $\vec{v}\times\vec{w} = \langle 0,0,0\rangle$, then $\vec{v} = \vec{w}$.
T	F	The curvature of a circle of radius r is equal to $1/(2\pi r)$.

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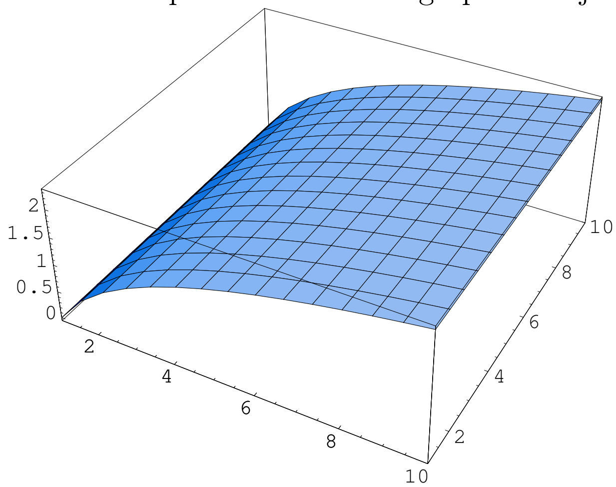
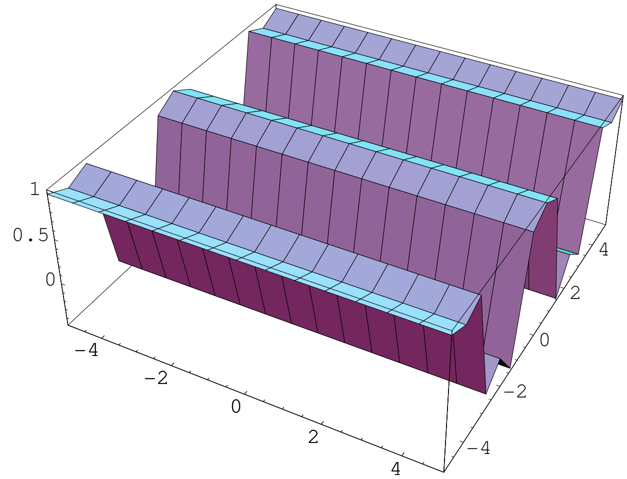
Problem 2) (10 points)

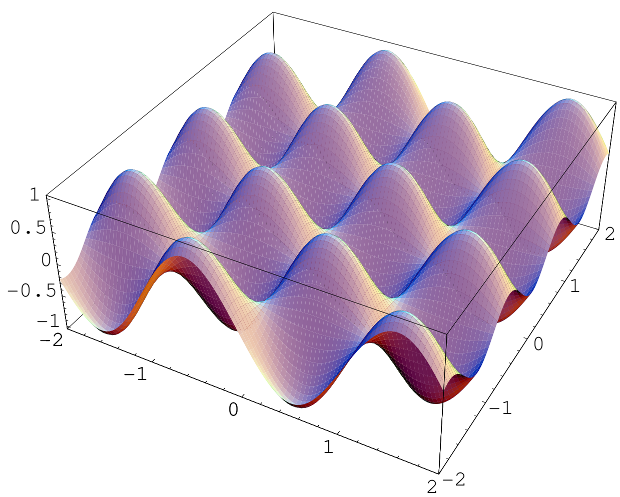
Match the equation with their graphs and justify briefly your choice.



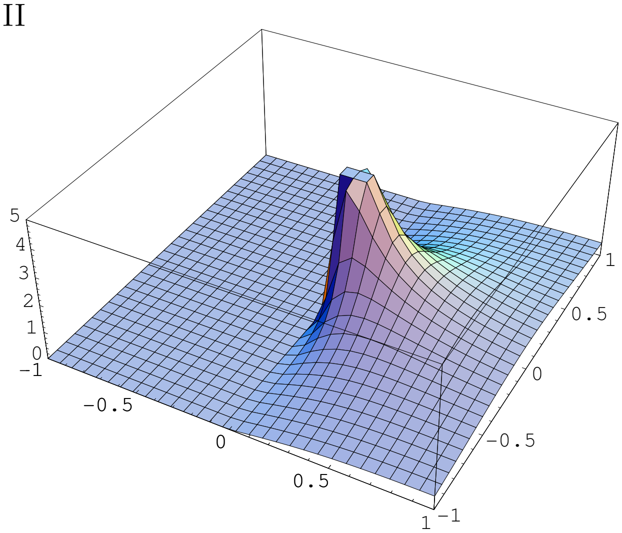
I



II



III

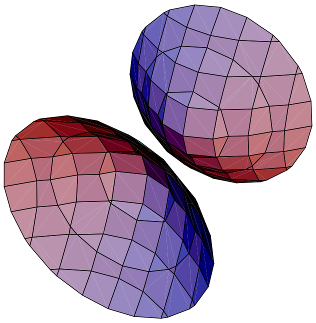


IV

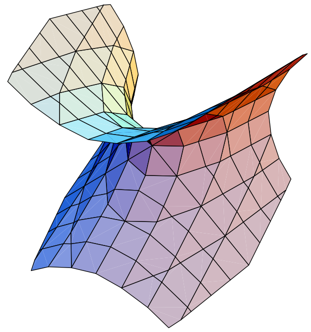
Enter I,II,III,IV here	Equation	Short Justification
	$z = \sin(3x)\cos(5y)$	
	$z = \cos(y^2)$	
	$z = \log(x)$	
	$z = x/(x^2 + y^2)$	

Problem 3) (10 points)

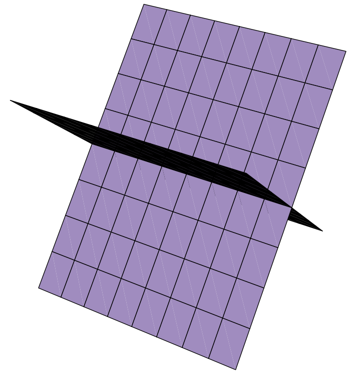
Match the equation with their graphs and justify briefly your choice.



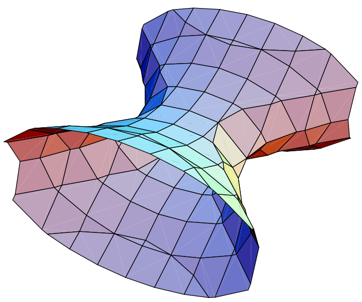
I



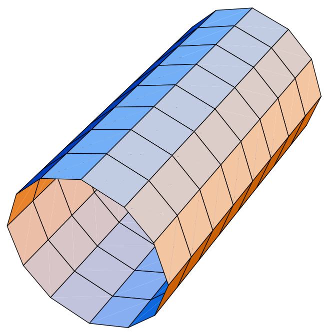
II



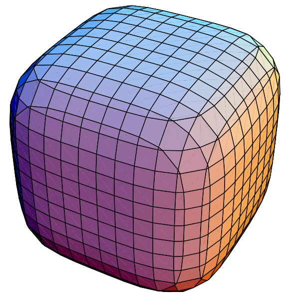
III



IV



V



VI

Enter I,II,III,IV,V,VI here	Equation	Short explanation
	$x^4 + y^4 + z^4 - 1 = 0$	
	$-x^2 + y^2 - z^2 - 1 = 0$	
	$x^2 + z^2 = 1$	
	$-y^2 + z^2 = 0$	
	$x^2 - y^2 + 3z^2 - 1 = 0$	
	$x^2 - y - z^2 = 0$	

Problem 4) (10 points)

Given the vectors $\vec{v} = \langle 1,1,0 \rangle$ and $\vec{w} = \langle 0,0,1 \rangle$ and the point $P = (2,4,-2)$. Let Σ be the plane which goes through the origin and contains the vectors $\vec{v}$ and $\vec{w}$.

a) Determine the distance from P to the origin.

b) Determine the distance from P to the plane Σ.

Problem 5) (10 points)

Boba Fett is flying through the air when his rocket pack malfunctions and sends him spinning out of control. At time $t = 0$, he is at the point $P_0 = (0,0,27)$ and moving with velocity $\vec{v} = \langle 10,0,0 \rangle$. While he is in the air, his acceleration is given by $\vec{a}(t) = \langle \pi^2 \sin \pi t, \pi^2 \cos \pi t + 2t, -6t \rangle$ for $t \geq 0$.

1. For $t \geq 0$, find Boba's position as a function of time.
2. The ground is represented by the xy plane. At what time does Boba hit the ground? What are the x and y coordinates of the point, where he hits?

Problem 6) (10 points)

a) Calculate the unit tangent vector T, the unit normal vector N as well as the binormal vector B for the curve $\vec{r}(t) = \langle t, \cos(t), t^2 \rangle$ at the point $t = \pi$.

b) Verify that for a general curve the formula $\frac{d}{dt}\vec{B}(t) = \vec{T}(t) \times \vec{N}'(t)$ holds.

Hint. Here are the formulas for the unit tangent vector, the unit normal vector as well as the binormal vector.

$$\vec{T}(t) = \vec{r}'(t)/|\vec{r}'(t)|$$
$$\vec{N}(t) = \vec{T}'(t)/|T'(t)|$$
$$\vec{B}(t) = \vec{T}(t) \times \vec{N}(t).$$

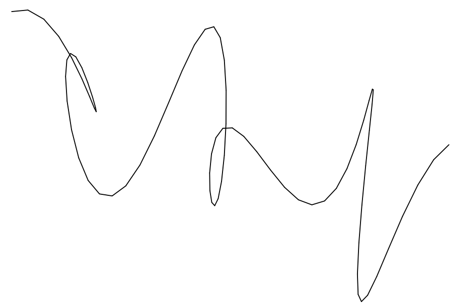
Problem 7) (10 points)

a) Identify the surface whose equation is given in spherical coordinates as $\theta = \pi/4$.

b) Identify the surface whose equation is given in spherical coordinates as $\phi = \pi/4$.

c) Identify the surface, whose equation is given in cylindrical coordinates by $z = r^2$. Either name it or sketch the surface convincingly.

Problem 8) (10 points)



Let $\vec{r}(t)$ be the space curve $\vec{r}(t) = (t^2, \sin(3\pi t), \cos(5\pi t))$.

a) Calculate the velocity, the acceleration and the speed of $\vec{r}(t)$ at time $t = 1$.

b) Write down the length of the curve from $t = 1$ to $t = 10$ as an integral. You don't have to evaluate the integral.

c) The curve $t \mapsto \vec{r}(t) = (t^3, 1 - t, 1 - t^3)$ lies in a plane. What is the equation of this plane?

Problem 9) (10 points)

Let S be the surface given by

$$z^2 = \frac{x^2}{4} + y^2 \ .$$

a) Sketch the surface S.

b) Let (a, b, c) be a point on the surface S. Find a parametric equation for the line that passes through (a, b, c) and lies entirely on the surface S.