Name:

- Start by printing your name in the above box and check your section in the box to the left.

| MWF10 Ken Chung |
| MWF10 Weiyang Qiu |
| MWF11 Oliver Knill |
| TTh10 Mark Lucianovic |
| TTh11.5 Ciprian Manolescu |

- Try to answer each question on the same page as the question is asked. If needed, use the back or the next empty page for work. If you need additional paper, write your name on it.

- Do not detach pages from this exam packet or un-staple the packet.

- Please write neatly. Answers which are illegible for the grader can not be given credit.

- No notes, books, calculators, computers, or other electronic aids can be allowed.

- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		100

Circle for each of the 20 questions the correct letter. No justifications are needed. Your score will be $C - W$ where C is the number of correct answers and W is the number of wrong answers.

T	F	
		The vector $\vec{v}$ connecting $(1, 2, 3)$ with $(4, 5, 3)$ is orthogonal to $\vec{w} = (0, 0, 3)$.

T	F	
		The length of the difference $\vec{v} - \vec{w}$ of two parallel vectors is the difference $|\vec{v}| - |\vec{w}|$ of the lengths of the vectors.

T	F	
		$\vec{k} = \vec{j} \times \vec{i}$.

T	F	
		The set of points in $\mathbf{R}^3$ which have distance 1 from a line form a cylinder.

T	F	
		The equation $x^2 - z^2 = y$ describes a one-sheeted hyperboloid.

T	F	
		If in rectangular coordinates, a point is given by $(1, 0, 1)$, then its spherical coordinates are $(\rho, \theta, \phi) = (\sqrt{2}, \pi/2, -\pi/2)$.

T	F	
		The curvature of a curve $\vec{r}(t)$ at $t = 0$ is the same as the curvature of the curve $\vec{r}(-t)$ at $t = 0$.

T	F	
		In spherical coordinates the equation $\cos(\theta) = \sin(\theta)$ defines the plane $x - y = 0$.

T	F	
		The velocity vector of a parametric curve $\vec{r}(t)$ always has length 1.

T	F	
		In spherical coordinates (ρ, θ, ϕ), the equations $\phi = \pi/2 = \rho$ define a circle.

T	F	
		For any three vectors $\vec{a}$, $\vec{b}$ and $\vec{c}$, we always have $(\vec{a} \times \vec{b}) \cdot \vec{c} = -(\vec{a} \times \vec{c}) \cdot \vec{b}$.

T	F	
		The set of points in the xy-plane which satisfy $x^2 - y^2 = -1$ is a hyperbola.

T	F	
		If $|\vec{v} \times \vec{w}| = 0$ then $\vec{v} = 0$ or $\vec{w} = 0$.

T	F	
		Two nonzero vectors are parallel if and only if their cross product is $\vec{0}$.

T	F	
		If the velocity vector $\vec{r}\,'(t)$ and the acceleration vector $\vec{r}\,''(t)$ of a curve are parallel then the curvature $\kappa(t)$ of the curve is zero.

T	F	
		The function $f(x, y) = \begin{cases} \frac{x^2 y^2}{x^2 + y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$ is discontinuous at $(0, 0)$.

T	F	
		The scalar projection of a vector $\vec{v}$ onto a vector $\vec{w}$ is always equal to the scalar projection of $\vec{w}$ onto $\vec{v}$.

| T	F	
		The graph of the function $f(x, y) = \cos(xy)$ can be written as a level surface of a function $g(x, y, z)$.

T	F	
		If the length of the velocity vector $\vec{r}\,'(t)$ does not depend on t then the curvature of the curve $\vec{r}(t)$ is zero.

T	F	
		The curvature of the curve $2\vec{r}(4t)$ at $t = 0$ is twice the curvature of the curve $\vec{r}(t)$ at $t = 0$.

$$\boxed{} = \boxed{}$$

Problem 2) (10 points)

Match the contour maps with the corresponding functions $f(x, y)$ of two variables. Note that one of the contour maps is not represented by a formula. No justifications are needed.
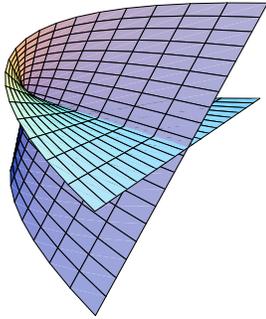
I

II

III

IV

V

VI
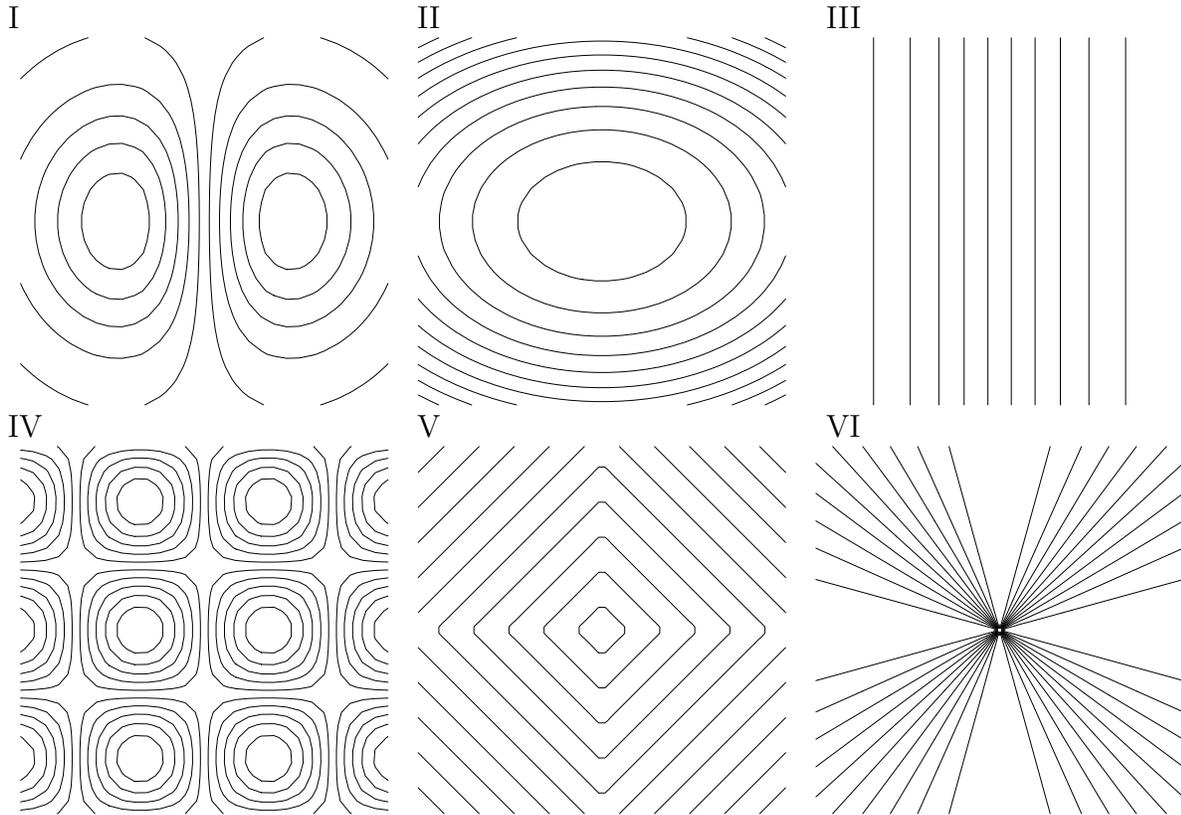
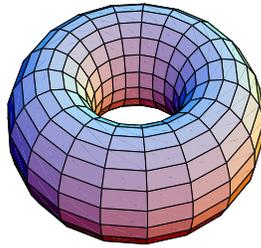| Enter I,II,III,IV,V or VI here	Function $f(x, y)$
	$f(x, y) = \sin(x)$
	$f(x, y) = x^2 + 2y^2$
	$f(x, y) = |x| + |y|$
	$f(x, y) = xe^{-x^2 - y^2}$
	$f(x, y) = x^2/(x^2 + y^2)$

Problem 3) (10 points)

Match the surfaces with their parameterization $\vec{r}(u,v)$ or equation $g(x,y,z) = 0$. Note that one of the surfaces is not represented by a formula. No justifications are needed.
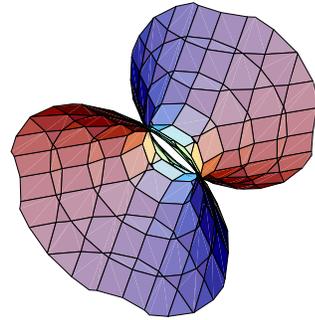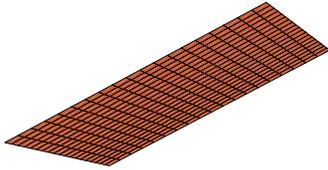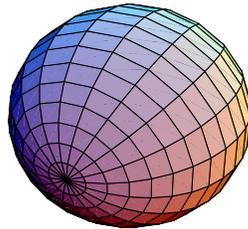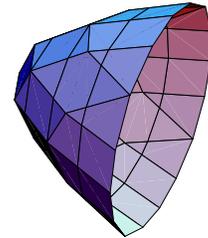
I

II

III

IV

V

VI

Enter I,II,III,IV,V,VI here	Equation or Parameterization
	$\vec{r}(u,v) = ((1+\sin(u))\cos(v), (1+\sin(u))\sin(v), \cos(u))$
	$\vec{r}(u,v) = (v, v-u, u+v)$
	$\vec{r}(u,v) = (u^2, vu, v)$
	$x^2 - y^2 + z^2 - 1 = 0$
	$\vec{r}(u,v) = (\cos(u)\sin(v), \cos(v), \sin(u)\sin(v))$

Problem 4) (10 points)

Find the distance between the two lines

$$\vec{r}_1(t) = (t, 2t, -t)$$

and

$$\vec{r}_2(t) = (1+t, t, t).$$

Problem 5) (10 points)

a) (6 points) Find a parameterization of the line of intersection of the planes $3x - 2y + z = 7$ and $x + 2y + 3z = -3$.

Hint. Use the fact that the line goes through the point $P = (1, -2, 0)$.

b) (4 points) Find the symmetric equations

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

representing that line.

Problem 6) (10 points)

Find the curvature $\kappa(t)$ of the space curve $\vec{r}(t) = (-\cos(t), \sin(t), -2t)$.

Hint. Use one of the two formulas for the curvature

$$\kappa(t) = \frac{|\vec{T}'(t)|}{|\vec{r}'(t)|} = \frac{|\vec{r}'(t) \times \vec{r}''(t)|}{|\vec{r}'(t)|^3} ,$$

where $\vec{T}(t) = \vec{r}'(t)/|\vec{r}'(t)|$.

Problem 7) (10 points)

The intersection of the two surfaces $x^2 + \frac{y^2}{2} = 1$ and $z^2 + \frac{y^2}{2} = 1$ consists of two curves.
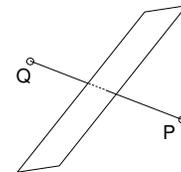
a) (4 points) Parameterize each curve in the form $\vec{r}(t) = (x(t), y(t), z(t))$.
b) (3 points) Set up the integral for the arc length of one of the curves.
c) (3 points) What is the arc length of this curve?

Problem 8) (10 points)

Find an equation $ax + by + cz = d$ for the plane which has the property that $Q = (5, 4, 5)$ is the reflection of $P = (1, 2, 3)$ through that plane.

Problem 9) (10 points)

Let S be the surface given in cylindrical coordinates as $r = 2 + \sin(z)$.

a) (5 points) Find a parameterization

$$\vec{r}(u, v) = (x(u, v), y(u, v), z(u, v))$$

of the surface.

b) (5 points) Sketch the surface S. Draw at least three grid curves for each parameter.