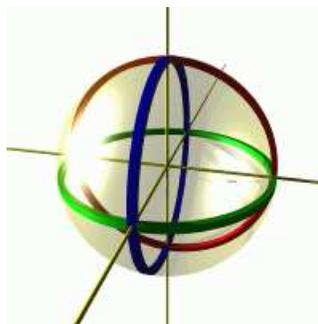


TRACES. To draw surfaces, it helps to look at the **traces**, the intersections of the surfaces with the coordinate planes $x = 0, y = 0$ or $z = 0$.

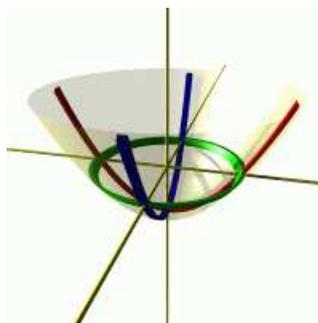
Quadrics: If $f(x, y, z) = ax^2 + by^2 + cz^2 + dxy + exz + fyz + gx + hy + kz + m$ then the surface $f(x, y, z) = 0$ is called a **quadric**. Below are some examples.

SPHERE.
All three traces are circles



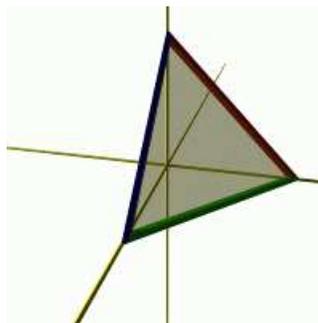
$$(x - a)^2 + (y - b)^2 + (z - c)^2 = r^2$$

PARABOLOID.
The z-traces are circles, the x and y traces are parabolas.



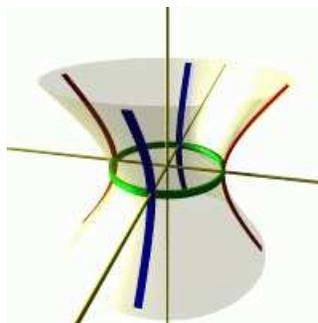
$$(x - a)^2 + (y - b)^2 - c = z$$

PLANE.
All three traces are lines.



$$ax + by + cz = d$$

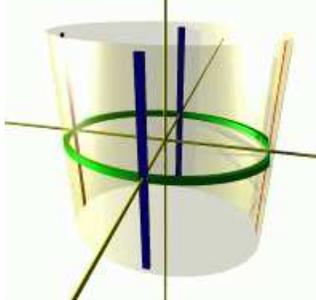
ONE SHEETED HYPERBOLOID.
The z-traces are circles, the x and y traces are hyperbolas.



$$(x - a)^2 + (y - b)^2 - (z - c)^2 = r^2$$

CYLINDER.

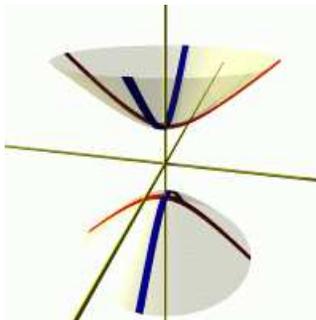
The z-traces are circles, the x and y traces are lines.



$$(x - a)^2 + (y - b)^2 = r^2$$

TWO SHEETED HYPERBOLOID.

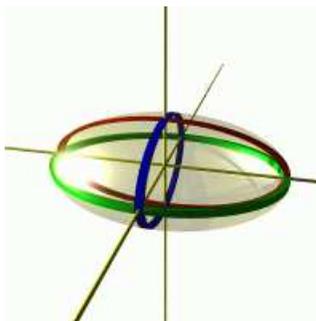
The z-traces are circles or empty, the x and y traces are hyperbolas.



$$(x - a)^2 + (y - b)^2 - (z - c)^2 = -r^2$$

ELLIPSOID.

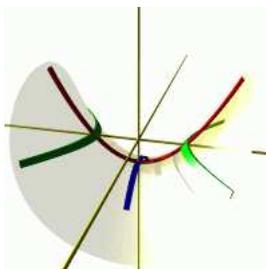
All three traces are ellipses.



$$x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$$

HYPERBOLIC PARABOLOID.

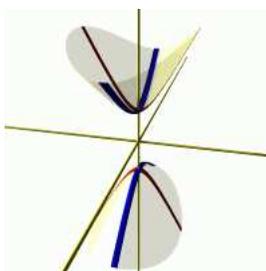
The z-trace form two crossed lines, the x- and y- traces are parabolas.



$$x^2 - y^2 + z = 1$$

TWO SHEETED ELLIPTIC HYPERBOLOID.

The z-traces are ellipses or empty, the x and y traces are hyperbolas.



$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = -r^2$$