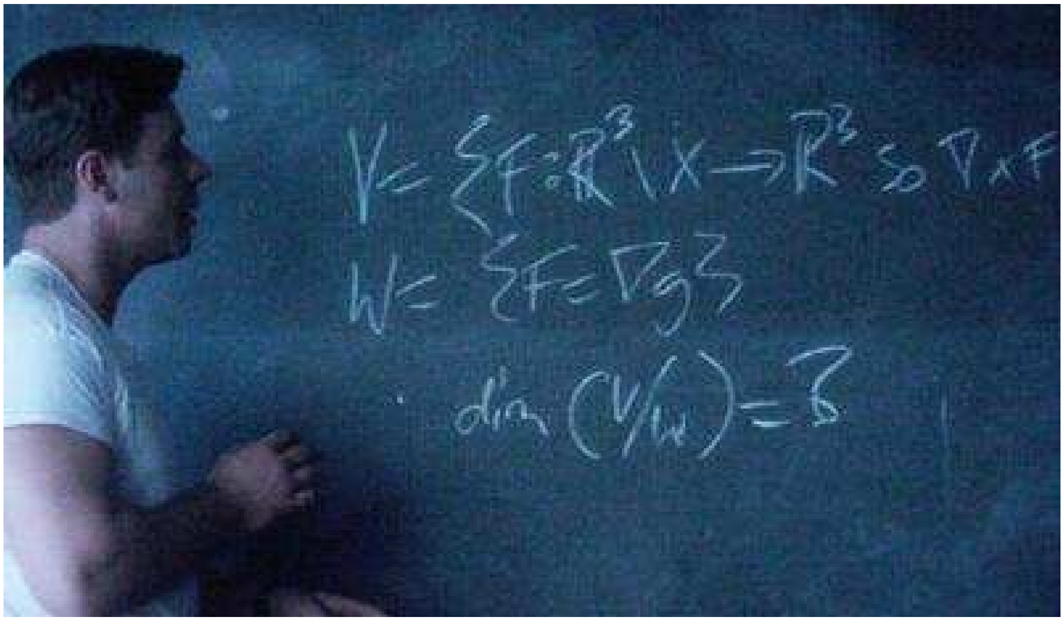
ICE: NASH'S PROBLEM **Math 21a**

In this ICE, you solve Nash's problem, he gave to a multivariable calculus class. Remember Nash saying in the movie "A beautiful mind":

> "It might take some of you a few months to solve it, for most of you however it might take a life time".



NASH's PROBLEM. Find a subset X of $\mathbf{R}^3$ with the property that if V is the set of vector fields F on $\mathbf{R}^3 \setminus X$ which satisfy curl$(F) = 0$ and W is the set of vector fields F which are conservative: $F = \nabla f$. Then, the space V/W should be 8 dimensional.

Remark. The meaning of the last sentence means that there should be 8 vectorfields F_i which are not gradient fields and which have vanishing curl outside X. (You might learn more about dimensions in Math21b, linear algebra.) Furthermore, you should not be able to write any of the 8 vectorfields as a sum of multiples of the other 7 vector fields.

You actually saw a two dimensional version of the problem in class:

2D VERSION OF NASH's PROBLEM.

If $X = \{0\}$ and V is the set of vector fields F on $\mathbf{R}^2 \setminus X$ which satisfy curl$(F) = 0$ and W is the set of vector fields F which are gradient fields, then $\dim(V/W) = 1$.

The vector field F is $F(x, y) = (-y/(x^2 + y^2), x/(x^2 + y^2))$ is a gradient field in $\mathbf{R}^2 \setminus X$ but not in $\mathbf{R}^2$.

Now: What set X would you have to take to get $\dim(V/W) = 8$?

The SOLUTION OF THE 3D VERSION OF NASH's PROBLEM can be obtained directly from the solution of the 2D version. How? This is a challenge problem.