THE VAN DER WAALS (1837-1923) EQUATION for real gases is

$$(p + \tfrac{a}{V^2})(V - b) = RT$$

where $R = 8.314 J/K$mol is a constant called the **Avogadro number**. This law was relates the **pressure** p, the **volume** V and the **temperature** T of a gas. The constant a is related to the molecular interactions, the constant b to the finite rest-volume of the molecules. For $a = b = 0$, it becomes the ideal gas law $PV = RT$. The **ideal gas** law $pV = RT$ is obtained when a, b are set to 0. The level curves or **isotherms** $T(V, p) = c$ tell much about the properties of the gas. The **reduced van der Walls law**

$$T(V, p) = (p + \tfrac{3}{V^2})(3V - 1)/8$$

is obtained by scaling p, T, V depending on the gas. It has the advantage that it does no more contain constants. We will work with this constant free version.

1) What is the gradient of $T(V, p)$ at $(1, 1)$? Check in the figure below whether the gradient is orthogonal to the level curve through $(1, 1)$.

2) Calculate the directional derivative of $T(V, p)$ at the point $(V, p) = (1, 1)$ into the direction $(3, 4)/5$.

3) Assume we expand and change the pressure of the gas according to $r(t) = (V(t), p(t)) = (2/5, 3/5) + (3t, 2t^2)/5$. The temperature at time t will be $T(t)$. What is $d/dt T(t)$ at time $t = 1$? Use the chain rule.