

**SURFACES OF REVOLUTION.** Consider a positive function  $g(x)$  on an interval  $[a, b]$  on the  $x$  axes and rotate the graph around the  $x$ -axes. We obtain a **surface of revolution** parameterized by  $(u, v) \mapsto \vec{r}(u, v) = (v, g(v) \cos(u), g(v) \sin(u))$  on  $R = [0, 2\pi] \times [a, b]$ .

**SURFACE AREA.** We have  $\vec{r}_u = (0, -g(v) \sin(u), g(v) \cos(u))$ ,  $\vec{r}_v = (1, g'(v) \cos(u), g'(v) \sin(u))$  and  $\vec{r}_u \times \vec{r}_v = (-g(v)g'(v), g(v) \cos(u), g(v) \sin(u)) = g(v)(-g'(v), \cos(u), \sin(u))$  which has the length  $|\vec{r}_u \times \vec{r}_v| = |g(v)|\sqrt{1 + g'(v)^2}$ . The surface area of such a surface of revolution is therefore

$$2\pi \int_a^b |g(v)|\sqrt{1 + g'(v)^2} dv.$$

**VOLUME.** If we cut through the surface perpendicular to the  $x$ -axes, we obtain a disc of radius  $g(x)$  and area  $\pi g(x)^2$ . The part of the surface between  $x$  and  $x + dx$  has volume  $g(x)^2 dx$ .

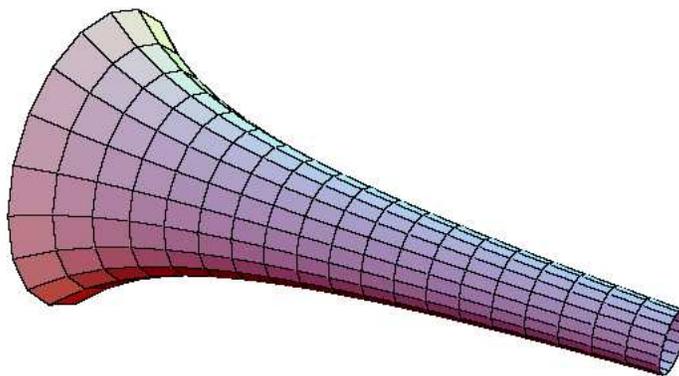
Therefore: The interior of a surface of revolution has volume  $\int_a^b \pi g(x)^2 dx$ .

**GABRIEL'S TRUMPET.** The surface of revolution defined by  $g(x) = 1/x$  on the interval  $[1, \infty)$  is called **Gabriel's trumpet**.

**Volume:** The volume is

**Surface area:** Fill in the rest: The surface area is

$$\geq 2\pi \int_1^\infty \frac{1}{x} dx = 2\pi \log(x)|_1^\infty = \infty.$$



We conclude: the Gabriel trumpet is a surface of finite volume but with infinite surface area! You can fill the trumpet with a finite amount of paint, but this paint does not suffice to cover the surface of the trumpet!