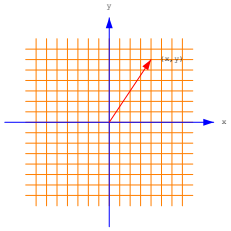
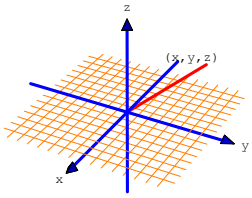
2/3/2003, COORDINATES/DISTANCES — O. Knill, Math 21a

Book: Multivariable calculus by J. Stewart
Web: www.courses.fas.harvard.edu/~math21a
QC: Su-Th, 8-10 PM Loker Commons

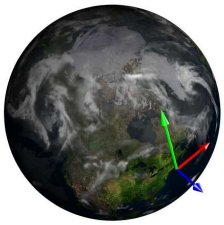
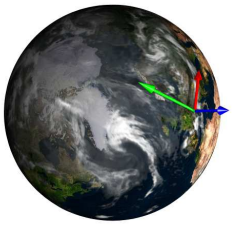
Homework: section 9.1, Problems: 2,8,10,14,34

CARTESIAN COORDINATE SYSTEMS. Points on the line are labeled by one coordinate x, a point in the **plane** is fixed by 2 coordinates (x, y) and points in space are determined by three coordinates (x, y, z).

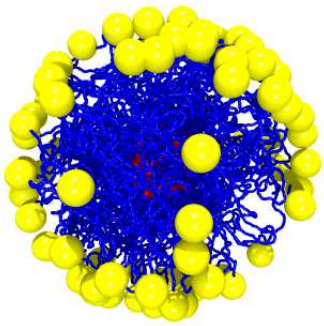
1D space = line =2 half lines 2D space = plane =4 quadrants 3D space = space =8 octants

CHOICE OF COORDINATE SYSTEM. Fixing the x,y or z-coordinate axes determines one of many possible coordinate systems. On earth for example, the z-axes usually points up. But this direction of course depends on the position on earth.

PHOTOGRAPHERS COORDINATE SYSTEM: in 3D graphics like computer games, virtual reality or ray tracing, it is custom to have the y-axis is up, the x-axis to the right and the z axis in front. This is the **"photographers coordinate system"**. If the photo is the x-y plane, then the depth is the z axes. Is the photographers coordinate system a left or right-handed coordinate system?



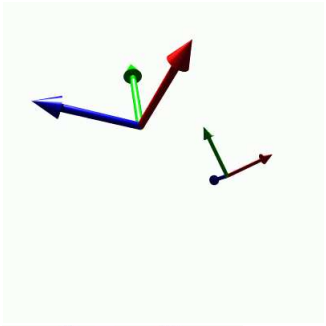
Z-BUFFER. In computer graphics, the part of the memory reserved for storing the z-axis is called the **"z-buffer"**. It is useful for "hidden line removal" in 2D rendering of a 3D scene: The z-axis is perpendicular to the screen with values increasing towards the viewer. Any point whose z-coordinate is less than the corresponding z-buffer value will be hidden behind feature already plotted.

DISTANCE. The distance between two points $P = (x, y, z)$ and $Q = (a, b, c)$ is

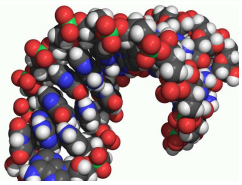
$$\boxed{d(P, Q) = \sqrt{(x-a)^2 + (y-b)^2 + (z-c)^2}}$$

(Pythagoras). While other distances could be defined, the Euclidean distance in the above formula is distinguished. Why?

PARITY. We usually work with a right handed coordinate system. The the "right hand rule": thumb=x-direction index finger=y-direction and middle finger=z-direction to check that the coordinate system is "right handed".



Parity plays a role in Biology (orientation of DNA or Proteins) or particle physics, ("parity violation": physical laws are not the same when we look at them in the mirror). Coordinate systems with different parity can not be rotated into each other.



GEOMETRICAL OBJECTS. **curves**, **surfaces** and **bodies** are examples of geometrical objects which can be described using **functions of several variables**. We look at some of them here to get some feel about space. The objects will be treated later in more detail.

SPHERE. The sphere is the collection of points which have a fixed distance r from a given point (a, b, c). The equation is $(x-a)^2+(y-b)^2+(z-c)^2 = r^2$.

COMPLETION OF SQUARE $x^2+bx+c = 0$. Solve by adding $(b/2)^2 - c$ on both sides (completion of square.

$$\begin{aligned} x^2+bx+c &= 0 \\ x^2+bx+(b/2)^2 &= (b/2)^2-c \\ (x+b/2)^2 &= (b/2)^2-c \\ x &= \pm\sqrt{(b/2)^2-c}-b/2\,. \end{aligned}$$

CENTER AND RADIUS OF CIRCLE.
The equation $x^2+5x+y^2-2y=-1$ is after completion of the square in each variable equivalent to $(x+5/2)^2 - 25/4+(y-1)^2-1=-1$ or $(x-5/2)^2+(y-1)^2=(5/2)^2$. The equation describes therefore a circle with center $(5/2, 1)$ and radius $5/2$.

COORDINATE PLANES, QUADRANTS, OCTANTS.
The coordinate axes $x = 0$, $y = 0$ divide the plane into 4 regions called quadrants. Similarly, the coordinate planes $x = 0$, $y = 0$ and $z = 0$ divide the space into 8 regions called octants. How many "hyper-regions" are there in four dimensional "hyper-space" which is labeled by points with 4 coordinates (t, x, y, z)? Give a point for each of these hyper-regions.

SOLUTION. There are 16 regions. Points are $(\pm 1, \pm 1, \pm 1 \pm 1)$

DESCRIBING PLANES.
We want to draw the set of all points (x, y, z) which satisfy $x + 2y - 3z = 2$.
a) One way to do this is to figure out, where the set intersects the x-coordinate axes, the y-coordinate axes and the z-coordinate axes and put a plane through these three points. Find these intersection points.
b) An other way to visualize the set is to find the traces, the intersections with the coordinate planes $x = 0$, $y = 0$ or $z = 0$. Find these traces.
c) Make a drawing of the plane which shows the intercepts found in a) and the traces found in b).

HISTORICAL. René Descartes (1596-1650) introduced in 1637 an appendix "Geometry" to his "Discours de la méthode". In it, he promoted the idea that algebra could be used as a general method to solve geometric problems. We call therefore the rectangular coordinate system also **Cartesian coordinate system**.

Annectote: "In 1649, Queen Christina of Sweden persuaded Descartes to go to Stockholm. However the Queen wanted to draw tangents at 5 a.m. and Descartes broke the habit of his lifetime of getting up at 11 o'clock. After only a few months in the cold northern climate, walking to the palace for 5 o'clock every morning, he died of pneumonia.