FIRST PRACTICE EXAM FOR FINAL **Math 21a, Spring 03**

Name:

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TTh11.5 Ciprian Manolescu

- Start by printing your name in the above box and check your section in the box to the left.
- Try to answer each question on the same page as the question is asked. If needed, use the back or next empty page for work. If you need additional paper, write your name on it.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader can not be given credit. Justify your answers.
- No notes, books, calculators, computers or other electronic aids are allowed.
- You have 180 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
11		10
12		10
13		10
Total:		140

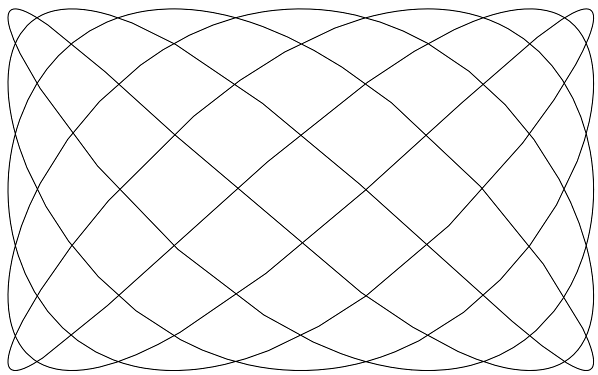
Problem 1) TF questions (20 points) Circle the correct letter. No justifications are needed.

T	F	The length of the curve $\mathbf{r}(t) = (\sin(t), t^4 + t, \cos(t))$ on $t \in [0,1]$ is the same as the length of the curve $\mathbf{r}(t) = (\sin(t^2), t^8 + t^2, \cos(t^2))$ on $[0,1]$.
T	F	The parametric surface $\mathbf{r}(u,v) = (5u - 3v, u - v - 1, 5u - v - 7)$ is a plane.
T	F	Any function $u(x,y)$ that obeys the differential equation $u_{xx} + u_x - u_y = 1$ has no local maxima.
T	F	The scalar projection of a vector $\mathbf{a}$ onto a vector $\mathbf{b}$ is the length of the vector projection of $\mathbf{a}$ onto $\mathbf{b}$.
T	F	If $f(x,y)$ is a function such that $f_x - f_y = 0$ then f is conservative.
T	F	$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w} = (\mathbf{u} \times \mathbf{w}) \cdot \mathbf{v}$ for all vectors $\mathbf{u}, \mathbf{v}, \mathbf{w}$.
T	F	The equation $\rho = \phi/4$ in spherical coordinates is half a cone.
T	F	The function $f(x,y) = \begin{cases} \frac{x}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$ is continuous at every point in the plane.
T	F	$\int_0^1 \int_0^x 1 \, dy dx = 1/2$.
T	F	Let $\mathbf{a}$ and $\mathbf{b}$ be two vectors which are perpendicular to a given plane Σ. Then $\mathbf{a} + \mathbf{b}$ is also perpendicular to Σ.
T	F	If $g(x,t) = f(x - vt)$ for some function f of one variable $f(z)$ then g satisfies the differential equation $g_{tt} - v^2 g_{xx} = 0$.
T	F	If $f(x,y)$ is a continuous function on $\mathbf{R}^2$ such that $\int\int_D f \, dA \geq 0$ for any region D then $f(x,y) \geq 0$ for all (x,y).
T	F	Assume the two functions $f(x,y)$ and $g(x,y)$ have both the critical point $(0,0)$ which are saddle points, then $f + g$ has a saddle point at $(0,0)$.
T	F	If $f(x,y)$ is a function of two variables and if $h(x,y) = f(g(y), g(x))$, then $h_x(x,y) = f_y(g(y), g(x)) g'(y)$.
T	F	If we rotate a line around the z axes, we obtain a cylinder.
T	F	The line integral of $\mathbf{F}(x,y) = (x,y)$ along an ellipse $x^2 + 2y^2 = 1$ is zero.
T	F	If $u(x,y)$ satisfies the transport equation $u_x = u_y$, then the vector field $\mathbf{F}(x,y) = \langle u(x,y), u(x,y) \rangle$ is a gradient field.
T	F	$3 \,\mathrm{grad}(f) = \frac{d}{dt} f(x+t, y+t, z+t)$.
T	F	$\int_0^1 \int_0^{2\pi/11} \int_0^{\pi} \rho^2 \sin(\phi) \, d\phi d\theta d\rho = 4\pi/33$.
T	F	If $\mathbf{F}$ is a vector field in space and f is equal to the line integral of $\mathbf{F}$ along the straight line C from $(0,0,0)$ to (x,y,z), then $\nabla f = \mathbf{F}$.

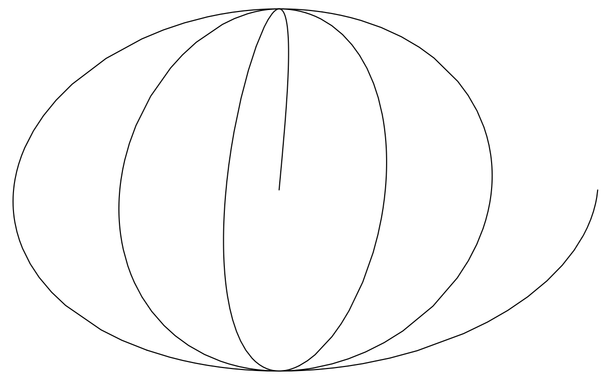
[ ] x 4 = [ ]

Problem 2) (10 points)

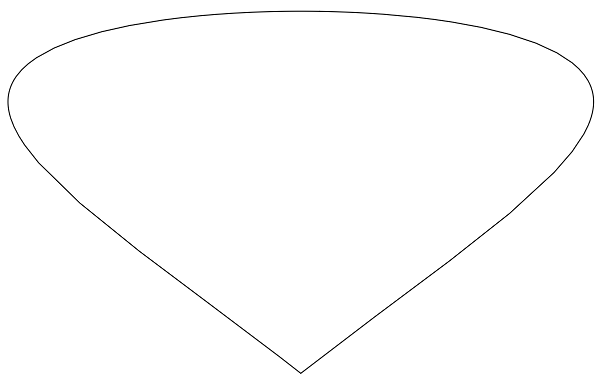
Match the equations with the curves. No justifications are needed.



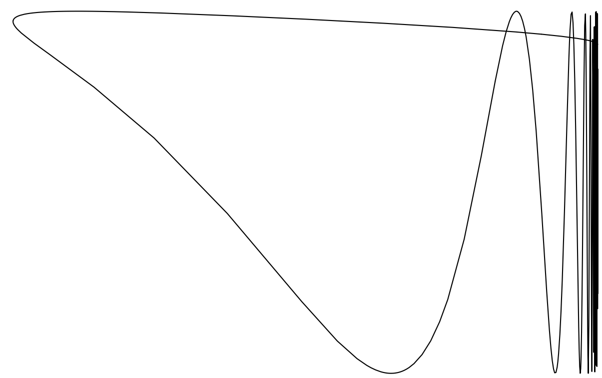
I



II



III



IV

Enter I,II,III,IV here	Equation
	$\mathbf{r}(t) = (\sin(t), t(2\pi - t))$
	$\mathbf{r}(t) = (\cos(5t), \sin(7t))$
	$\mathbf{r}(t) = (t\cos(t), \sin(t))$
	$\mathbf{r}(t) = (\cos(t), \sin(6/t))$

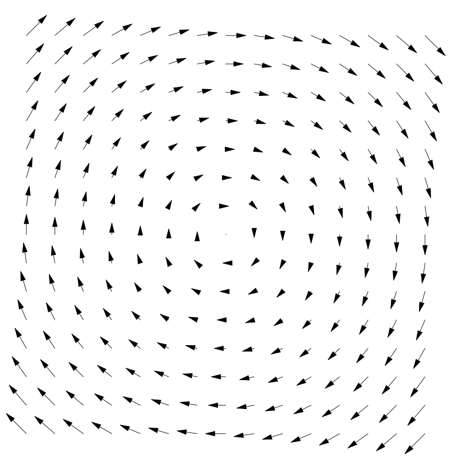
Problem 3) (10 points)

In this problem, vector fields F are written as $F = (P, Q)$. We use abbreviations $\text{curl}(F) = Q_x - P_y$ and $\text{div}(F) = P_x + Q_y$. When stating $\text{curl}(F)(x, y) = 0$ we mean that $\text{curl}(F)(x, y) = 0$ vanishes for **all** (x, y). The statement $\text{curl}(F) \neq 0$ means that $\text{curl}(F)(x, y)$ does not vanish for at least one point (x, y). The same remark applies if curl is replaced by div.

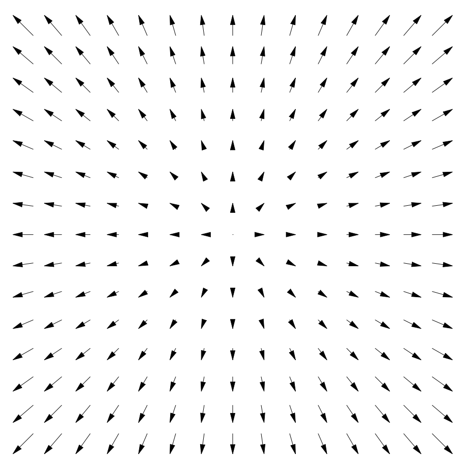
Check the box which match the formulas of the vectorfields with the corresponding picture I,II,III or IV. Mark also the places, indicating the vanishing or not vanishing of curl and div. In each of the four lines, you should finally have circled three boxes. No justifications are needed.

Vectorfield	I	II	III	IV	$\text{curl}(F) = 0$	$\text{curl}(F) \neq 0$	$\text{div}(F) = 0$	$\text{div}(F) \neq 0$
$\mathbf{F}(x, y) = (0, 5)$								
$\mathbf{F}(x, y) = (y, -x)$								
$\mathbf{F}(x, y) = (x, y)$								
$\mathbf{F}(x, y) = (2, x)$								

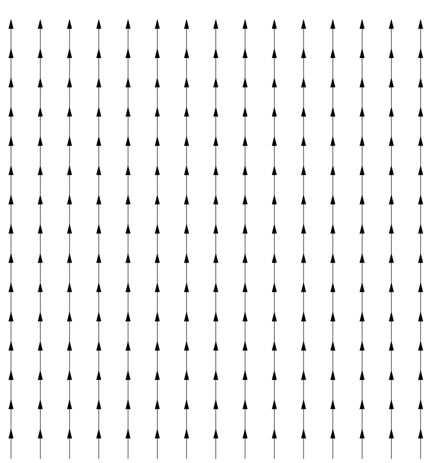
I



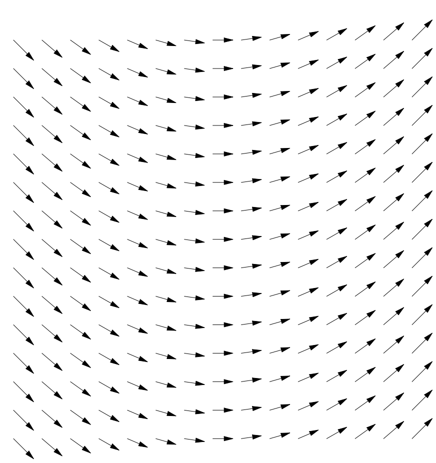
II



III



IV



Problem 4) (10 points)

a) Find the scalar projection of the vector $\mathbf{v} = (3, 4, 5)$ onto the vector $\mathbf{w} = (2, 2, 1)$.

b) Find the equation of a plane which contains the vectors $\langle 1, 1, 0\rangle$ and $\langle 0, 1, 1\rangle$ and contains the point $(0, 1, 0)$.

Problem 5) (10 points)

Find the surface area of the ellipse cut from the plane $z = 2x+2y+1$ by the cylinder $x^2+y^2 = 1$.

Problem 6) (10 points)

Sketch the plane curve $\mathbf{r}(t) = (\sin(t)e^t, \cos(t)e^t)$ for $t \in [0, 2\pi]$ and find its length.

Problem 7) (10 points)

Let $f(x, y, z) = 2x^2+3xy+2y^2+z^2$ and let R denote the region in $\mathbf{R}^3$, where $2x^2+2y^2+z^2 \leq 1$. Find the maximum and minimum values of f on the region R and list all points, where said maximum and minimum values are achieved. Distinguish between local extrema in the interior and extrema on the boundary.

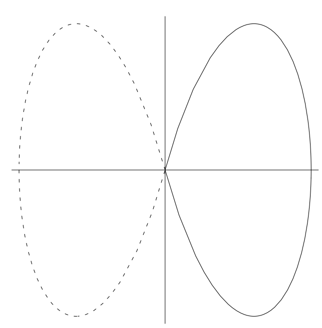
Problem 8) (10 points)

Sketch the region of integration of the following iterated integral and then evaluate the integral:

$$\int_0^{\pi} \left(\int_{\sqrt{z}}^{\sqrt{\pi}} \left(\int_0^x \sin(xy)dy \right) dx \right) dz \,.$$

Problem 9) (10 points)

Evaluate the line integral $\int_C \mathbf{F} \cdot dr$, where $\mathbf{F} = (x + e^x \sin(y), x + e^x \cos(y))$ and C is the right handed loop of the lemniscate described in polar coordinates as $r^2 = \cos(2\theta)$.



Problem 10) (10 points)

Evaluate the line integral

$$\int_C \mathbf{F} \cdot dr,$$

where C is the planar curve $\mathbf{r}(t) = (t^2, t/\sqrt{t+2}), t \in [0, 2]$ and $\mathbf{F}$ is the vector field $\mathbf{F}(x, y) = (2xy, x^2 + y)$. Do this in two different ways:

a) by verifying that $\mathbf{F}$ is conservative and replacing the path with a different path connecting $(0, 0)$ with $(4, 1)$,

b) by finding a potential U satisfying $\nabla U = \mathbf{F}$.

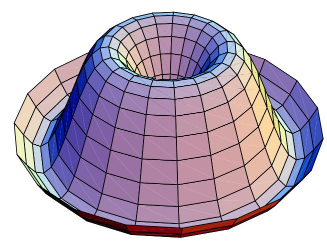
Problem 11) (10 points)

a) Find the line integral $\int_C \mathbf{F} \cdot dr$ of the vector field $\mathbf{F}(x, y) = (xy, x)$ along the unit circle $C : t \mapsto \mathbf{r}(t) = (\cos(t), \sin(t)), t \in [0, 2\pi]$ by doing the actual line integral.

b) Find the value of the line integral obtained in a) by evaluating a double integral.

Problem 12) (10 points)

Consider the surface given by the graph of the function $z = f(x, y) = \frac{100}{1+x^2+y^2} \sin\left(\frac{\pi}{8}(x^2 + y^2)\right)$ in the region $x^2 + y^2 \leq 16$. The surface is pictured to the right.



A magnetic field $\mathbf{B}$ is given by the curl of a vector potential $\mathbf{A}$. That is, $\mathbf{B} = \nabla \times \mathbf{A} = \text{curl}(A)$ and $\mathbf{A}$ is a vector field too. Suppose

$$\mathbf{A} = \left(z \sin(x^3), x\left(1 - z^2\right), \log(1 + e^{x+y+z})\right) .$$

Compute the flux of the magnetic field through this surface. (The surface has an upward pointing normal vector.)

Problem 13) (10 points)

Let S be the surface given by the equations $z = x^2 - y^2$, $x^2 + y^2 \leq 4$, with the upward pointing normal. If the vector field $\mathbf{F}$ is given by the formula $\mathbf{F}(x, y, z) = \langle -x, y, \sqrt{x^2 + y^2} \rangle$, find the flux of $\mathbf{F}$ through S.