Name:

- Start by printing your name in the above box and check your section in the box to the left.

- Try to answer each question on the same page as the question is asked. If needed, use the back or next empty page for work. If you need additional paper, write your name on it.

| MWF10 Ken Chung |
| MWF10 Weiyang Qiu |
| MWF11 Oliver Knill |
| TTh10 Mark Lucianovic |
| TTh11.5 Ciprian Manolescu |

- Do not detach pages from this exam packet or un-staple the packet.

- Please write neatly. Answers which are illegible for the grader can not be given credit. Justify your answers and give details unless stated otherwise.

- No notes, books, calculators, computers or other electronic aids are allowed.

- You have 180 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
11		10
12		10
13		10
Total:		140

Problem 1) TF questions (20 points) Circle the correct letter. Your score is the number of correct answers. No justifications are needed.

| T	F	For any two nonzero vectors $\vec{v}, \vec{w}$ the vector $((\vec{v} \times \vec{w}) \times \vec{v}) \times \vec{v}$ is parallel to $\vec{w}$.
T	F	The cross product satisfies the law $(\vec{u} \times \vec{v}) \times \vec{w} = \vec{u} \times (\vec{v} \times \vec{w})$.
T	F	If the curvature of a smooth curve $\vec{r}(t)$ in space is defined and zero for all t, then the curve is part of a line.
T	F	The curve $\vec{r}(t) = (1-t)A + tB, t \in [0,1]$ connects the point A with the point B.
T	F	For every c, the function $u(x,t) = (2\cos(ct) + 3\sin(ct))\sin(x)$ is a solution to the wave equation $u_{tt} = c^2 u_{xx}$.
T	F	The length of the curve $\vec{r}(t) = (t, \sin(t))$, where $t \in [0, 2\pi]$ is $\int_0^{2\pi} \sqrt{1 + \cos^2(t)}\, dt$.
T	F	Let (x_0, y_0) be the maximum of $f(x,y)$ under the constraint $g(x,y) = 1$. Then $f_{xx}(x_0, y_0) < 0$.
T	F	The function $f(x,y,z) = x^2 - y^2 - z^2$ decreases in the direction $(2, -2, -2)/\sqrt{8}$ at the point $(1,1,1)$.
T	F	Assume $\vec{F}$ is a vector field satisfying $|\vec{F}(x,y,z)| \le 1$ everywhere. For every curve $C : \vec{r}(t)$ with $t \in [0,1]$, the line integral $\int_C \vec{F} \cdot d\vec{r}$ is less or equal than the arc length of C.
T	F	Let $\vec{F}$ be a vector field which coincides with the unit normal vector $\vec{N}$ for each point on a curve C. Then $\int_C \vec{F} \cdot d\vec{r} = 0$.
T	F	The divergence of the gradient of any $f(x,y,z)$ is always zero.
T	F	For every function f, one has $\text{div}(\text{curl}(\text{grad}(f))) = 0$.
T	F	If for two vector fields $\vec{F}$ and $\vec{G}$ one has $\text{curl}(\vec{F}) = \text{curl}(\vec{G})$, then $\vec{F} = \vec{G} + (a,b,c)$, where a, b, c are constants.
T	F	For every vector field $\vec{F}$ the identity $\text{grad}(\text{div}(\vec{F})) = \vec{0}$ holds.
T	F	If a nonempty quadric surface $g(x,y,z) = ax^2 + by^2 + cz^2 = 5$ can be contained inside a finite box, then $a, b, c \ge 0$.
T	F	If $\vec{F}$ is a vector field in space then the flux of $\vec{F}$ through any closed surface S is 0.
T	F	If $\text{div}(\vec{F})(x,y,z) = 0$ for all (x,y,z), then $\text{curl}(\vec{F}) = (0,0,0)$ for all (x,y,z).
T	F	The flux of the vector field $\vec{F}(x,y,z) = (y+z, y, -z)$ through the boundary of a solid region E is equal to the volume of E.
T	F	If in spherical coordinates the equation $\phi = \alpha$ (with a constant α) defines a plane, then $\alpha = \pi/2$.
T	F	For every function $f(x,y,z)$, there exists a vector field $\vec{F}$ such that $\text{div}(\vec{F}) = f$.

Match the equations with the objects. No justifications are needed.

I   II   III   IV
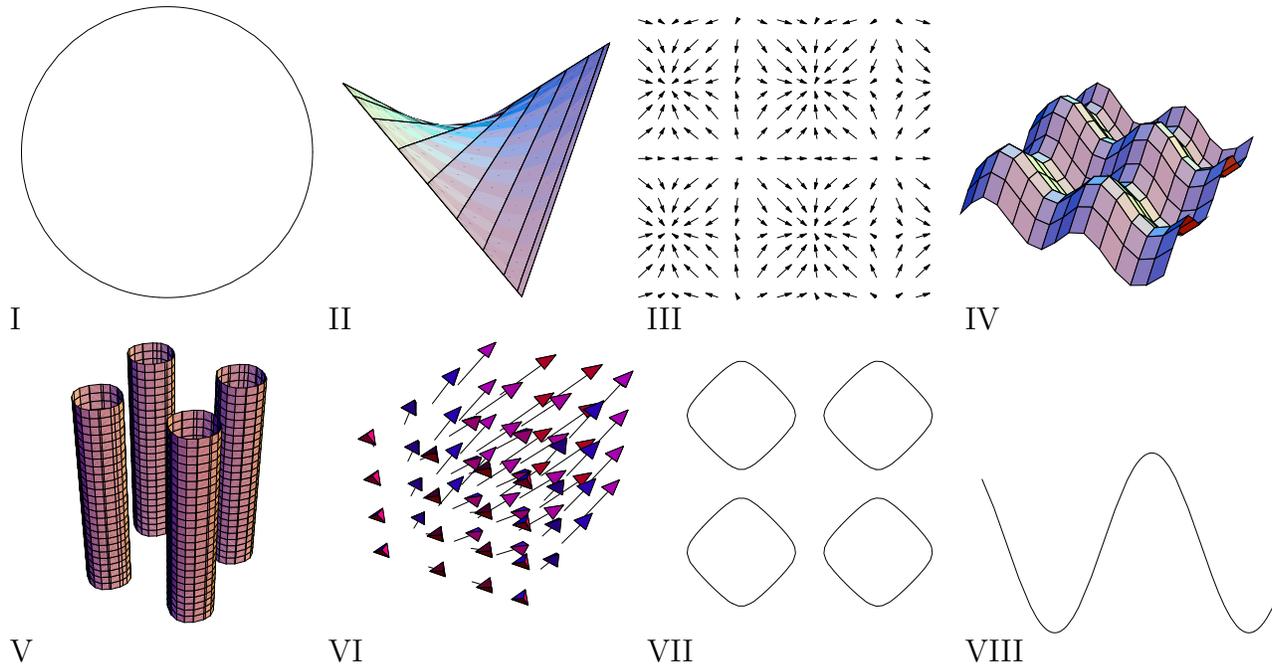
V   VI   VII   VIII

Enter I,II,III,IV,V,VI,VII,VIII here	Equation
	$g(x, y, z) = \cos(x) + \sin(y) = 1$
	$y = \cos(x) - \sin(x)$
	$\vec{r}(t) = (\cos(t), \sin(t))$
	$\vec{r}(u, v) = (\cos(u), \sin(v), \cos(u)\sin(v))$
	$\vec{F}(x, y, z) = (\cos(x), \sin(x), 1)$
	$z = f(x, y) = \cos(x) + \sin(y)$
	$g(x, y) = \cos(x) - \sin(y) = 1$
	$\vec{F}(x, y) = (\cos(x), \sin(x))$

Problem 3) (10 points)

Mark with a cross in the column below "conservative" if a vector fields is conservative (that is if $\text{curl}(\vec{F})(x, y, z) = (0, 0, 0)$ for all points (x, y, z)). Similarly, mark the fields which are incompressible (that is if $\text{div}(\vec{F})(x, y, z) = 0$ for all (x, y, z)). No justifications are needed.

Vectorfield	conservative $\text{curl}(\vec{F}) = \vec{0}$	incompressible $\text{div}(\vec{F}) = 0$
$\vec{F}(x, y, z) = (-5, 5, 3)$		
$\vec{F}(x, y, z) = (x, y, z)$		
$\vec{F}(x, y, z) = (-y, x, z)$		
$\vec{F}(x, y, z) = (x^2 + y^2, xyz, x - y + z)$		
$\vec{F}(x, y, z) = (x - 2yz, y - 2zx, z - 2xy)$		

Problem 4) (10 points)

Let E be a parallelogram in three dimensional space defined by two vectors $\vec{u}$ and $\vec{v}$.

a) (3 points) Express the diagonals of the parallelogram as vectors in terms of $\vec{u}$ and $\vec{v}$.

b) (3 points) What is the relation between the length of the crossproduct of the diagonals and the area of the parallelogram?

c) (4 points) Assume that the diagonals are perpendicular. What is the relation between the lengths of the sides of the parallelogram?

Problem 5) (10 points)

Find the volume of the largest rectangular box with sides parallel to the coordinate planes that can be inscribed in the ellipsoid $\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{25} = 1$.

Problem 6) (10 points)

Evaluate
$$\int_0^8 \int_{y^{1/3}}^2 \frac{y^2 e^{x^2}}{x^8} \, dx \, dy.$$

Problem 7) (10 points)

In this problem we evaluate $\int \int_D \frac{(x-y)^4}{(x+y)^4} \, dxdy$, where D is the triangular region bounded by the x and y axes and the line $x + y = 1$.

a) (3 points) Find the region R in the uv-plane which is transformed into D by the change of variables $u = x - y, v = x + y$. (It is enough to draw a carefully labeled picture of R.)

b) (3 points) Find the Jacobian $\frac{\partial(x,y)}{\partial(u,v)}$ of the transformation $(x, y) = (\frac{u+v}{2}, \frac{v-u}{2})$.

c) (4 points) Evaluate $\int \int_D \frac{(x-y)^4}{(x+y)^4} \, dxdy$ using the above defined change of variables.

Problem 8) (10 points)

a) (3 points) Find all the critical points of the function $f(x, y) = -(x^4 - 8x^2 + y^2 + 1)$.

b) (3 points) Classify the critical points.

c) (2 points) Locate the local and absolute maxima of f.

d) (2 points) Find the equation for the tangent plane to the graph of f at each absolute maximum.

Problem 9) (10 points)

Find the volume of the wedge shaped solid that lies above the xy plane and below the plane $z = x$ and within the cylinder $x^2 + y^2 = 4$.

Problem 10) (10 points)

Let the curve C be parametrized by $\vec{r}(t) = (t, \sin t, t^2 \cos t)$ for $0 \le t \le \pi$. Let $f(x, y, z) = z^2 e^{x+2y} + x^2$ and $\vec{F} = \nabla f$.

Find $\int_C \vec{F} \cdot d\vec{r}$.

Problem 11) (10 points)

Evaluate the line integral of the vector field $\vec{F}(x, y) = (y^2, x^2)$ in the clockwise direction around the triangle in the xy-plane defined by the points $(0, 0), (1, 0)$ and $(1, 1)$ in two ways:

a) (5 points) by evaluating the three line integrals.

b) (5 points) using Greens theorem.

Use Stokes theorem to evaluate the line integral of $\vec{F}(x, y, z) = (-y^3, x^3, -z^3)$ along the curve $\vec{r}(t) = (\cos(t), \sin(t), 1 - \cos(t) - \sin(t))$ with $t \in [0, 2\pi]$.

Let S be the graph of the function $f(x, y) = 2 - x^2 - y^2$ which lies above the disk $\{(x, y) \mid x^2 + y^2 \leq 1\}$ in the xy-plane. The surface S is oriented so that the normal vector points upwards. Compute the flux $\int \int_S \vec{F} \cdot d\vec{S}$ of the vectorfield

$$\vec{F} = (-4x + \frac{x^2 + y^2 - 1}{1 + 3y^2}, 3y, 7 - z - \frac{2xz}{1 + 3y^2})$$

through S using the divergence theorem.