10/2/2012 FIRST HOURLY PRACTICE III **Math 21a, Fall 2012**

Name:

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- Start by printing your name in the above box and **check your section** in the box to the left.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader cannot be given credit.
- **Show your work**. Except for problems 1-2 and 8, we need to see details of your computation.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		110

Problem 1) True/False (TF) questions (20 points)

Mark for each of the 20 questions the correct letter. No justifications are needed.

1) T F The surface $x^2 + y^2 + z^2 + 2z = 0$ is a sphere.

2) T F The length of the vector $\langle 1, 2, 2 \rangle$ is an integer.

3) T F The vector $\langle 3, 4 \rangle$ appears as a velocity vector of the curve $\vec{r}(t) = \langle \cos(5t), \sin(5t) \rangle$. Namely, there is a t such that $\vec{r}\,'(t) = \langle 3, 4 \rangle$.

4) T F If $\vec{T}$ is the unit tangent vector, $\vec{N}$ is the unit normal vector, and $\vec{B}$ is the binormal vector, then $\vec{B} \times \vec{N} = \vec{T}$.

5) T F The curvature of a larger circle $r = 2$ is greater than the curvature of a smaller circle $r = 1/2$.

6) T F The surface $x^2 - y^2 - z^2 - 1 = 0$ is a one sheeted hyperboloid.

7) T F The function $f(x, y) = y^2 - x^2$ has a graph that is an elliptic paraboloid.

8) T F Let $\vec{r}(t)$ be a parametrization of a curve. If $\vec{r}(t)$ is always parallel to the tangent vector $\vec{r}\,'(t)$, then the curve is part of a line through the origin.

9) T F If $\text{proj}_{\vec{k}}(\vec{u})$ is perpendicular to $\vec{u}$, then $\vec{u}$ is the zero vector.

10) T F If $\text{proj}_{\vec{k}}(\vec{u})$ is perpendicular to $\vec{u}$, then $\text{proj}_{\vec{k}}(\vec{u})$ is the zero vector.

11) T F If $\vec{u} \times \vec{v} = \vec{0}$ then $\vec{u} = \vec{0}$ or $\vec{v} = \vec{0}$.

12) T F There are two vectors $\vec{a}$ and $\vec{b}$ such that the scalar projection of $\vec{a}$ onto $\vec{b}$ is 100 times the magnitude of $\vec{b}$.

13) T F The curve $\vec{r}(t) = \langle \cos(t), e^t + 10, t^2 \rangle, 2 \leq t \leq 6$ and the curve $\vec{r}(t) = \langle \cos(2t), e^{2t}, 4t^2 \rangle, 1 \leq t \leq 3$ have the same length.

14) T F The equation $\rho \sin(\phi) - 2\sin(\theta) = 0$ in spherical coordinates defines a two sheeted hyperboloid.

15) T F If triple scalar product of three vectors $\vec{u}, \vec{v}, \vec{w}$ is larger than $|\vec{u} \times \vec{v}|$ then $|\vec{w}| > 1$.

16) T F The distance between the x-axis and the line $x = y = 1$ is $\sqrt{2}$.

17) T F The vector $\langle -1, 2, 3 \rangle$ is perpendicular to the plane $x - 2y - 3z = 9$.

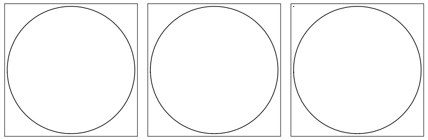
18) T F The curve $\vec{r}(t) = t^3 \langle 1, 2, 3 \rangle$ is a line.

19) T F The point $(1, 1, -\sqrt{3})$ is in spherical coordinates given by $(\rho, \theta, \phi) = (\sqrt{5}, \pi/4, 2\pi/3)$.

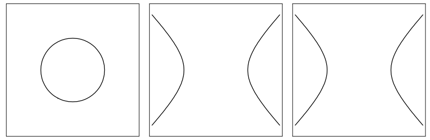
20) T F If the cross product satisfies $(\vec{v} \times \vec{w}) \times \vec{v} = \vec{0}$ then $\vec{v}$ and $\vec{w}$ are orthogonal.

Problem 2a) (6 points)

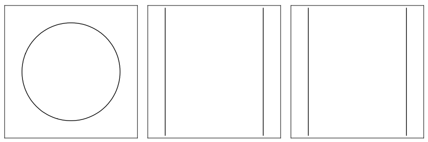
xy-trace yz-trace xz-trace xy-trace yz-trace xz-trace



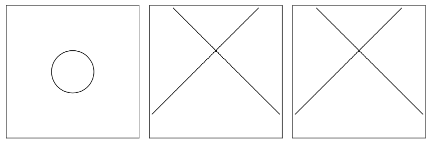
A



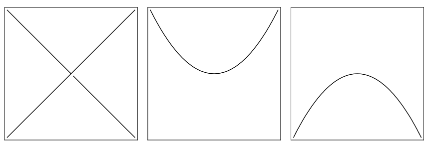
B



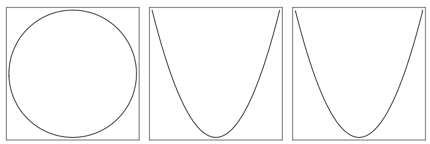
C



D



E

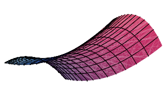


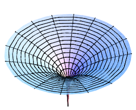
F

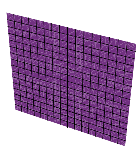
The figures above show the xy-trace, (the intersection of the surface with the xy-plane), the yz-trace (the intersection of the surface with the yz-plane), and the xz-trace (the intersection of the surface with the xz-plane). Match the following equations with the traces. No justifications required.

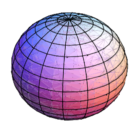
Enter A,B,C,D,E,F here	Equation
	$x^2 + y^2 - (z - 1/3)^2 = 0$
	$x^2 + y^2 + z^2 - 1 = 0$
	$x^2 - y^2 - z = 0$
	$x^2 + y^2 - 1 = 0$
	$x^2 + y^2 - z^2 - 1 = 0$
	$x^2 + y^2 - z = 1$

Problem 2b) (4 points)









I II III IV

Match the parametric surfaces with their parameterization. No justifications are needed.

Enter I,II,III,IV here	Parameterization
	$\vec{r}(u,v) = \langle u^2, v^2, u^4 - v^4 \rangle$
	$\vec{r}(u,v) = \langle \cos(u)\sin(v), 1 + \sin(u)\sin(v), \cos(v) \rangle$
	$\vec{r}(u,v) = \langle v\cos(u), v\sin(u), v^{1/4} \rangle$
	$\vec{r}(u,v) = \langle u, 3, v \rangle$

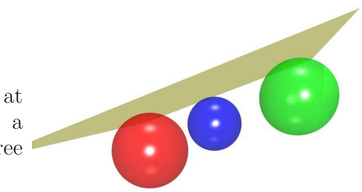
Problem 3) (10 points)

Find the distance of the point $P = (3,4,5)$ to the line

$$\frac{x-1}{4} = \frac{y-2}{5} = \frac{z-3}{6} .$$

Problem 4) (10 points)

Given three spheres of radius 1 centered at $A = (1,2,0), B = (4,5,0), C = (1,3,2)$. Find a plane $ax + by + cz = d$ which touches each of three spheres from the same side.



Problem 5) (10 points)

Find the arc length of the curve

$$\vec{r}(t) = \langle t^3/3, t^4/2, 2t^5/5 \rangle$$

from $0 \leq t \leq 1$.

Problem 6) (10 points)

An apple at position $(0, 0, 20)$ rests 20 meters above Newton's head, the tip of whose nose is at $(1, 0, 0)$. The apple falls with constant acceleration $\vec{r}\,''(t) = \langle a, 0, -10 \rangle$ (where $\langle 0, 0, -10 \rangle$ is caused by gravity and $\langle a, 0, 0 \rangle$ by the wind) precisely onto the nose of Newton. Find the wind force $\langle a, 0, 0 \rangle$ which achieves this. Give a parametrization for the path along which the apple falls.

Problem 7) (10 points)

a) (5 points) A red maple leaf falls to the ground $z = 0$. It falls along the curve $\vec{r}(t) = \langle 3\sqrt{3}\cos(t), 3\sqrt{3}\sin(t), 5-t-4t^2 \rangle$. At which angle does it hit the xy-plane?

b) (5 points) Find the tangent line to the curve at the impact point.

Problem 8) (10 points)

a) (5 points) The surface

$$\vec{r}(t, s) = \langle 1 + t + s, 1 - t - 2s, 1 + t - s \rangle$$

with $0 \leq t \leq 1, 0 \leq s \leq 1$ is a parallelogram in space. Find the area of this parallelogram.

b) (5 points) Another surface is given in spherical coordinates by $\rho = 2\sin(\phi)\cos(\theta)$. Write down the equation of this surface in rectangular coordinates as well as in cylindrical coordinates.

Problem 9) (10 points)

a) (5 points) Parametrize the curve obtained by intersecting the surface $z - x^2 + y^3 = 0$ with the cylindrical surface $x^2/4 + 9y^2 = 1$.

b) (5 points) Find the unit tangent vector $\vec{T}$ and the normal vector $\vec{N}(t) = \vec{T}\,'(t)/|\vec{T}\,'(t)|$ to the curve

$$\vec{r}(t) = \langle 3, t^2, t \rangle$$

at the point $(3, 0, 0)$. What is the binormal vector $\vec{B} = \vec{T} \times \vec{N}$?

Problem 10) (10 points)

a) (4 points) Give a parametrization of the hyperboloid $x^2 + y^2 = z^2 + 1$.
b) (3 points) Give a parametrization of the plane $x + y = 1$.
c) (3 points) Give a parametrization of the ellipsoid $x^2 + y^2 + z^2/4 = 1$.