

Name:

MWF 9 Chung-Jun John Tsai
MWF 10 Ivana Bozic
MWF 10 Peter Garfield
MWF 10 Oliver Knill
MWF 11 Peter Garfield
MWF 11 Stefan Hornet
MWF 12 Aleksander Subotic
TTH 10 Ana Caraiani
TTH 10 Toby Gee
TTH 10 Valentino Tosatti
TTH 11:30 Ming-Tao Chuan
TTH 11:30 Valentino Tosatti

- Start by printing your name in the above box and check your section in the box to the left.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- **Show your work.** Except for problems 1-2, we need to see details of your computation.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.

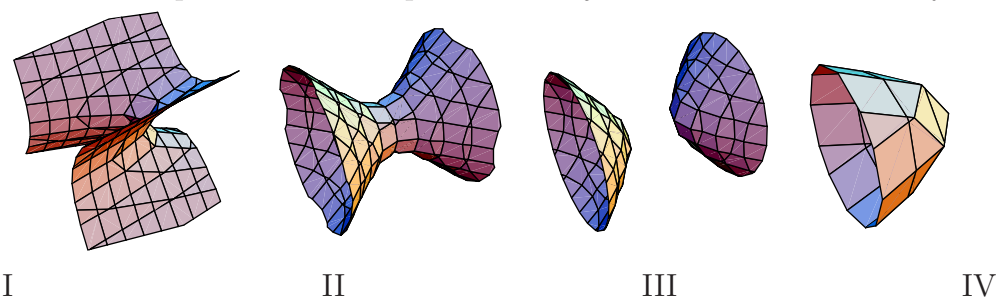
1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Total:		110

Mark for each of the 20 questions the correct letter. No justifications are needed.

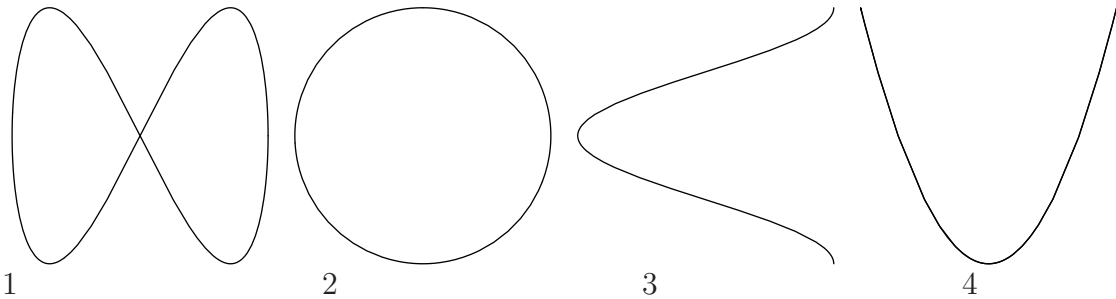
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|-----|---|---|
| 1) | <div style="display: inline-block; border: 1px solid black; padding: 2px 10px; margin: 0 5px;">T</div> <div style="display: inline-block; border: 1px solid black; padding: 2px 10px; margin: 0 5px;">F</div> | For a moving frame $(\vec{T}, \vec{N}, \vec{B})$, (remember that $\vec{T}$ is the unit tangent vector, $\vec{N}$ is the normal vector and $\vec{B}$ is the binormal vector), one always has $\vec{B} \cdot (\vec{T} \times \vec{N}) = 1$. |
| 2) | <div style="display: inline-block; border: 1px solid black; padding: 2px 10px; margin: 0 5px;">T</div> <div style="display: inline-block; border: 1px solid black; padding: 2px 10px; margin: 0 5px;">F</div> | For any three points P, Q, R in space, $\vec{PQ} \times \vec{PR} = \vec{QP} \times \vec{RP}$ |
| 3) | <div style="display: inline-block; border: 1px solid black; padding: 2px 10px; margin: 0 5px;">T</div> <div style="display: inline-block; border: 1px solid black; padding: 2px 10px; margin: 0 5px;">F</div> | The triangle defined by the three points $(-1, 0, 2), (-4, 2, 1), (1, -1, 2)$ has a right angle. |
| 4) | <div style="display: inline-block; border: 1px solid black; padding: 2px 10px; margin: 0 5px;">T</div> <div style="display: inline-block; border: 1px solid black; padding: 2px 10px; margin: 0 5px;">F</div> | The function $f(x, y, z) = x^2 + y^2 + z^2$ satisfies the partial differential equation $f_x^2 + f_y^2 + f_z^2 = 6f$. |
| 5) | <div style="display: inline-block; border: 1px solid black; padding: 2px 10px; margin: 0 5px;">T</div> <div style="display: inline-block; border: 1px solid black; padding: 2px 10px; margin: 0 5px;">F</div> | $\vec{u} \times \vec{u} = 0$ implies $\vec{u} = \vec{0}$. |
| 6) | <div style="display: inline-block; border: 1px solid black; padding: 2px 10px; margin: 0 5px;">T</div> <div style="display: inline-block; border: 1px solid black; padding: 2px 10px; margin: 0 5px;">F</div> | The level curves $f(x, y) = 1$ and $f(x, y) = 2$ of a smooth function f never intersect. |
| 7) | <div style="display: inline-block; border: 1px solid black; padding: 2px 10px; margin: 0 5px;">T</div> <div style="display: inline-block; border: 1px solid black; padding: 2px 10px; margin: 0 5px;">F</div> | For any vector $\vec{v}$, we have $proj_{\vec{i}}(proj_{\vec{j}}(\vec{v})) = \vec{0}$. |
| 8) | <div style="display: inline-block; border: 1px solid black; padding: 2px 10px; margin: 0 5px;">T</div> <div style="display: inline-block; border: 1px solid black; padding: 2px 10px; margin: 0 5px;">F</div> | $(\vec{j} \times \vec{i}) \times \vec{i} = \vec{k} \times (\vec{i} \times \vec{k})$ |
| 9) | <div style="display: inline-block; border: 1px solid black; padding: 2px 10px; margin: 0 5px;">T</div> <div style="display: inline-block; border: 1px solid black; padding: 2px 10px; margin: 0 5px;">F</div> | If a parametrized curve $\vec{r}(t)$ lies in a plane and the velocity $\vec{r}'(t)$ is never zero, then the normal vector $\vec{N}(t)$ also lies in that plane. |
| 10) | <div style="display: inline-block; border: 1px solid black; padding: 2px 10px; margin: 0 5px;">T</div> <div style="display: inline-block; border: 1px solid black; padding: 2px 10px; margin: 0 5px;">F</div> | The angle between $\vec{r}'(t)$ and $\vec{r}''(t)$ is always 90 degrees. |
| 11) | <div style="display: inline-block; border: 1px solid black; padding: 2px 10px; margin: 0 5px;">T</div> <div style="display: inline-block; border: 1px solid black; padding: 2px 10px; margin: 0 5px;">F</div> | If $\vec{v}, \vec{w}$ are two nonzero vectors, then the projection vector $proj_{\vec{w}}(\vec{v})$ can be longer than $\vec{v}$. |
| 12) | <div style="display: inline-block; border: 1px solid black; padding: 2px 10px; margin: 0 5px;">T</div> <div style="display: inline-block; border: 1px solid black; padding: 2px 10px; margin: 0 5px;">F</div> | A line intersects an ellipsoid in at most 2 distinct points. |
| 13) | <div style="display: inline-block; border: 1px solid black; padding: 2px 10px; margin: 0 5px;">T</div> <div style="display: inline-block; border: 1px solid black; padding: 2px 10px; margin: 0 5px;">F</div> | For any vectors $\vec{v}$ and $\vec{w}$, the formula $(\vec{v} - \vec{w}) \cdot \vec{P}_{\vec{w}}(\vec{v}) = 0$ holds. |
| 14) | <div style="display: inline-block; border: 1px solid black; padding: 2px 10px; margin: 0 5px;">T</div> <div style="display: inline-block; border: 1px solid black; padding: 2px 10px; margin: 0 5px;">F</div> | Let S be a plane normal to the vector $\vec{n}$, and let P and Q be points not on S . If $\vec{n} \cdot \vec{PQ} = 0$, then P and Q lie on the same side of S . |
| 15) | <div style="display: inline-block; border: 1px solid black; padding: 2px 10px; margin: 0 5px;">T</div> <div style="display: inline-block; border: 1px solid black; padding: 2px 10px; margin: 0 5px;">F</div> | The vectors $\langle 2, 2, 1 \rangle$ and $\langle 1, 1, -4 \rangle$ are perpendicular. |
| 16) | <div style="display: inline-block; border: 1px solid black; padding: 2px 10px; margin: 0 5px;">T</div> <div style="display: inline-block; border: 1px solid black; padding: 2px 10px; margin: 0 5px;">F</div> | $\ \vec{v} \times \vec{w}\ = \ \vec{v}\ \ \vec{w}\ \cos(\alpha)$, where α is the angle between $\vec{v}$ and $\vec{w}$. |
| 17) | <div style="display: inline-block; border: 1px solid black; padding: 2px 10px; margin: 0 5px;">T</div> <div style="display: inline-block; border: 1px solid black; padding: 2px 10px; margin: 0 5px;">F</div> | The vector $\vec{i} \times (\vec{j} \times \vec{k})$ has length 1. |
| 18) | <div style="display: inline-block; border: 1px solid black; padding: 2px 10px; margin: 0 5px;">T</div> <div style="display: inline-block; border: 1px solid black; padding: 2px 10px; margin: 0 5px;">F</div> | The distance between the z -axis and the line $x - 1 = y = 0$ is 1. |
| 19) | <div style="display: inline-block; border: 1px solid black; padding: 2px 10px; margin: 0 5px;">T</div> <div style="display: inline-block; border: 1px solid black; padding: 2px 10px; margin: 0 5px;">F</div> | There is a quadric surface which both hyperbola and parabola appear as traces. Traces are intersections of the surface with the coordinate planes $x = 0$, $y = 0$, or $z = 0$. |
| 20) | <div style="display: inline-block; border: 1px solid black; padding: 2px 10px; margin: 0 5px;">T</div> <div style="display: inline-block; border: 1px solid black; padding: 2px 10px; margin: 0 5px;">F</div> | The equation $x^2 + y^2 - z^2 = -1$ defines a one-sheeted hyperboloid. |

Problem 2) (10 points)

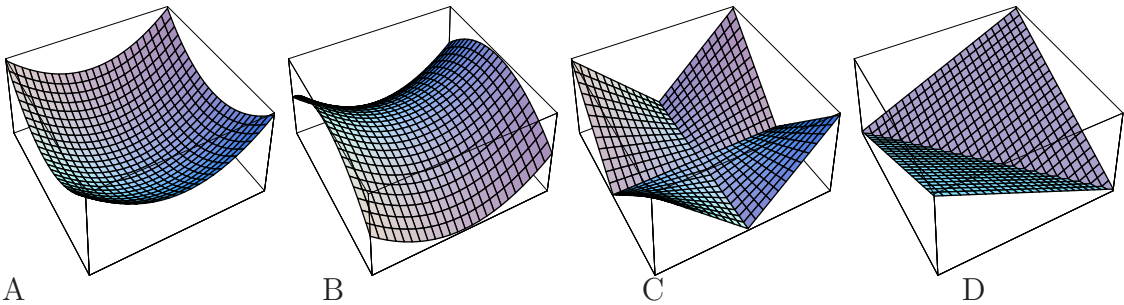
Match the equation with the pictures. No justifications are necessary in this problem.



Enter I,II,III,IV here	Equation
	$x + y^2 - z^2 - 1 = 0$
	$-x^2 + y^2 + z^2 - 1 = 0$
	$-x^2 + y^2 + z^2 + 1 = 0$
	$-x + y^2 + z^2 + 1 = 0$



Enter 1,2,3,4 here	Equation
	$\langle \cos(t), \sin(t) \rangle$
	$\langle \cos(t), t \rangle$
	$\langle \cos(t), \cos^2(t) \rangle$
	$\langle \cos(t), \sin(2t) \rangle$



Enter A,B,C,D here	Equation
	$f(x, y) = x^2 - y^2$
	$f(x, y) = x + y $
	$f(x, y) = x^2 + y^2$
	$f(x, y) = xy $

Problem 3) (10 points)

Imagine the planet Earth as the unit sphere in 3D space centered at the origin. An asteroid is approaching from the point $P = (0, 4, 3)$ along the path

$$\vec{r}(t) = \langle (4 - t) \sin(t), (4 - t) \cos(t), 3 - t \rangle .$$

- a) When and where will it first hit the Earth?
- b) What velocity will it have at the impact?



Problem 4) (10 points)

Find the distance between the cylinder $x^2 + y^2 = 1$ and the line

$$L : \frac{x + 2}{4} = \frac{y - 1}{3} = \frac{z}{2}.$$

Problem 5) (10 points)

- a) Find a parametrization $\vec{r}(t)$ of the line which is the intersection of the two planes

$$4x + 6y - z = 1$$

and

$$4x + z = 0 .$$

- b) Find the point on the line which is closest to the origin.

Problem 6) (10 points)

Consider the parameterized curve

$$\vec{r}(t) = \langle e^t + e^{-t}, 2 \cos(t), 2 \sin(t) \rangle .$$

Find the arc length of this curve from $t = 0$ to $t = 4$.

Problem 7) (10 points)

The set of points P for which the distance from P to $A = (1, 2, 3)$ is equal to the distance from P to $B = (5, 8, 5)$ forms a plane S .

- a) Find the equation $ax + by + cz = d$ of the plane S .
- b) Find the distance from A to S .

Problem 8) (10 points)

The Swiss tennis player Roger Federer hits the ball at the point $\vec{r}(0) = (0, 0, 3)$. The initial velocity is $\vec{r}'(0) = \langle 100, 10, 13 \rangle$. The tennis ball experiences a constant acceleration $\vec{r}''(t) = \langle 2, 0, -32 \rangle$ which is due to the combined force of gravity and a constant wind in the x direction.



- a) Where does the tennis ball hit the ground $z = 0$?
- b) What is the z -component = (projection onto z vector) $\text{proj}_{\vec{k}}(\vec{r}'(t))$ of the ball velocity at the impact?

Problem 9) (10 points)

- a) (4 points) Parameterize the intersection of the ellipsoid

$$\frac{x^2}{4} + \frac{(y-5)^2}{4} + \frac{z^2}{9} = 2$$

with the plane $z = 3$.

- b) (3 points) Parametrize the ellipsoid itself in the form

$$\vec{r}(\theta, \phi) = \dots$$

- c) (3 points) What is the curvature of the curve at the point $(2, 0, 3)$?

Hint. While you can use the curvature formula $\kappa(t) = \frac{|\vec{r}'(t) \times \vec{r}''(t)|}{|\vec{r}'(t)|^3}$ you are also allowed to cite a fact which you know about the curvature.

Problem 10) (10 points)

Find an equation $ax + by + cz = d$ for the plane which has the property that $Q = (5, 4, 5)$ is the reflection of $P = (1, 2, 3)$ through that plane.

