

Name:

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| MWF 10 Sug Woo Shin |
| MWF 10 Jay Pottharst |
| MWF 11 Oliver Knill |
| MWF 11 Kai-Wen Lan |
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| TTH 10 Ilia Zharkov |
| TTH 11 David Harvey |
| TTH 11 Ilia Zharkov |

- Start by printing your name in the above box and check your section in the box to the left.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which are illegible for the grader can not be given credit.
- No notes, books, calculators, computers, or other electronic aids can be allowed.
- You have 90 minutes time to complete your work.
- The hourly exam itself will have space for work on each page. This space is excluded here in order to save printing resources.

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|--------|--|-----|
| 1 | | 20 |
| 2 | | 10 |
| 3 | | 10 |
| 4 | | 10 |
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| 7 | | 10 |
| 8 | | 10 |
| 9 | | 10 |
| 10 | | 10 |
| Total: | | 110 |

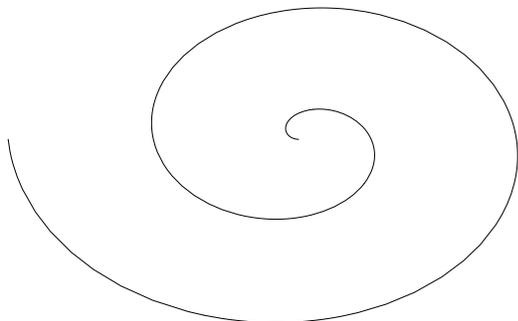
Problem 1) TF questions (20 points) No justifications needed

- 1) T F The length of the sum of two vectors is always the sum of the length of the vectors.
- 2) T F For any three vectors, $\vec{v} \times (\vec{w} + \vec{u}) = \vec{w} \times \vec{v} + \vec{u} \times \vec{v}$.
- 3) T F The set of points which satisfy $x^2 + 2x + y^2 - z^2 = 0$ is a cone.
- 4) T F The functions $\sqrt{x + y - 1}$ and $\log(x + y - 1)$ have the same domain of definition.
- 5) T F If P, Q, R are 3 different points in space that don't lie in a line, then $\vec{PQ} \times \vec{RQ}$ is a vector orthogonal to the plane containing P, Q, R .
- 6) T F The line $\vec{r}(t) = \langle 1 + 2t, 1 + 3t, 1 + 4t \rangle$ hits the plane $2x + 3y + 4z = 9$ at a right angle.
- 7) T F The graph of $f(x, y) = \cos(xy)$ is a level surface of a function $g(x, y, z)$.
- 8) T F For any two vectors, $\vec{v} \times \vec{w} = \vec{w} \times \vec{v}$.
- 9) T F If $|\vec{v} \times \vec{w}| = 0$ for all vectors \vec{w} , then $\vec{v} = \vec{0}$.
- 10) T F If \vec{u} and \vec{v} are orthogonal vectors, then $(\vec{u} \times \vec{v}) \times \vec{u}$ is parallel to \vec{v} .
- 11) T F Every vector contained in the line $\vec{r}(t) = \langle 1 + 2t, 1 + 3t, 1 + 4t \rangle$ is parallel to the vector $\langle 1, 1, 1 \rangle$.
- 12) T F There is a quadric $ax^2 + by^2 + cz^2 + dx + ey + fz = e$ which is a hyperbola when intersected with the plane $z = 0$, which is a hyperbola when intersected with the plane $y = 0$ and which is a parabola when intersected with $x = 0$.
- 13) T F The curvature of the curve $2\vec{r}(4t)$ at $t = 0$ is twice the curvature of the curve $\vec{r}(t)$ at $t = 0$.
- 14) T F The set of points which satisfy $x^2 - 2y^2 - 3z^2 = 0$ form an ellipsoid.
- 15) T F If $\vec{v} \times \vec{w} = (0, 0, 0)$, then $\vec{v} = \vec{w}$.
- 16) T F Every vector contained in the line $\vec{r}(t) = \langle 1 + 2t, 1 + 3t, 1 + 4t \rangle$ is parallel to the vector $\langle 1, 1, 1 \rangle$.
- 17) T F Two nonzero vectors are parallel if and only if their cross product is $\vec{0}$.
- 18) T F The vector $\vec{u} \times (\vec{v} \times \vec{w})$ is always in the same plane together with \vec{v} and \vec{w} .
- 19) T F The line $\vec{r}(t) = \langle 1 + 2t, 1 + 2t, 1 - 4t \rangle$ hits the plane $x + y + z = 9$ at a right angle.
- 20) T F The intersection of the ellipsoid $x^2/3 + y^2/4 + z^2/3 = 1$ with the plane $y = 1$ is a circle.

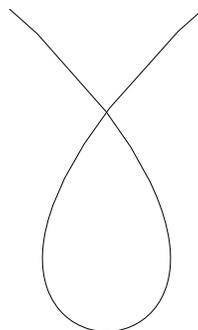
Problem 2a) (10 points)

Match the cruves with their parametric definitions.

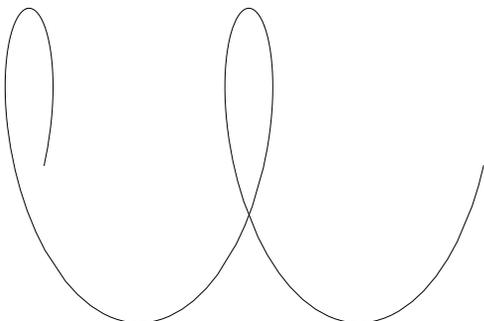
I



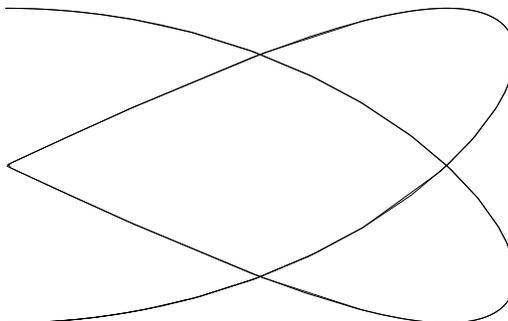
II



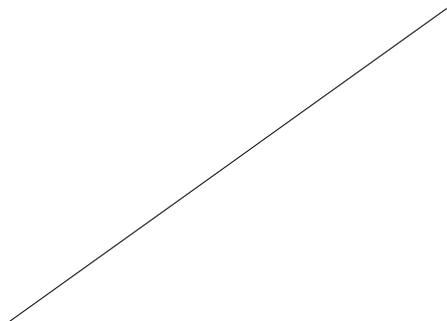
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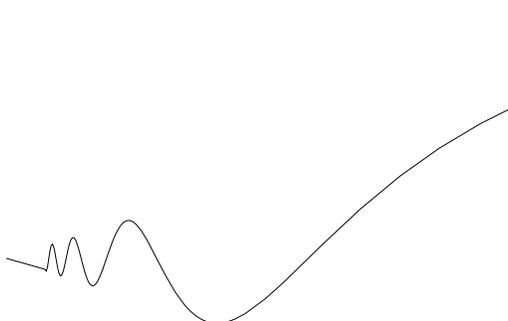
IV



V



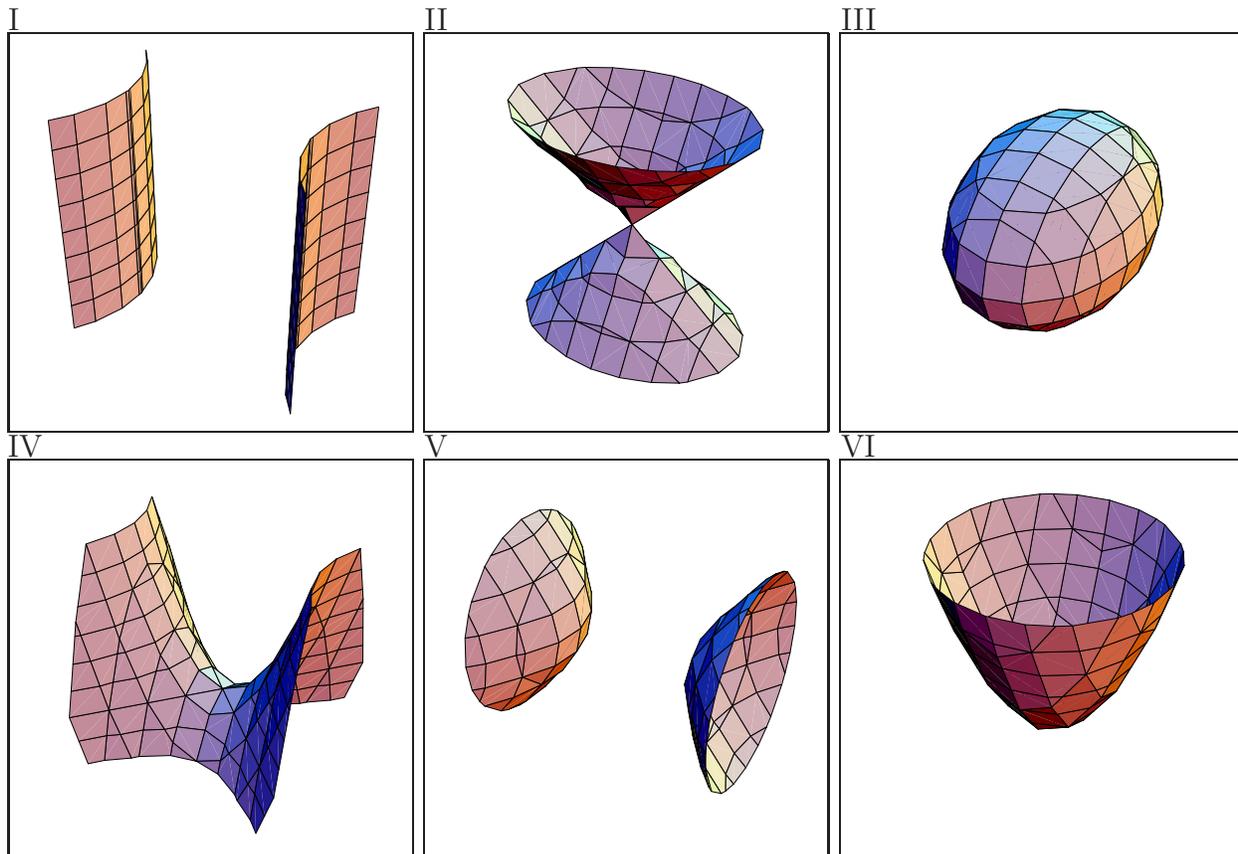
VI



| Enter I,II,III,IV,V or VI here | Parametric equation for the curve |
|--------------------------------|---|
| | $\vec{r}(t) = \langle t, \sin(1/t)t \rangle$ |
| | $\vec{r}(t) = \langle t^3 - t, t^2 \rangle$ |
| | $\vec{r}(t) = \langle t + \cos(2t), \sin(2t) \rangle$ |
| | $\vec{r}(t) = \langle \sin(2t) , \cos(3t) \rangle$ |
| | $\vec{r}(t) = \langle 1 + t, 5 + 3t \rangle$ |
| | $\vec{r}(t) = \langle -t \cos(t), 2t \sin(t) \rangle$ |

Problem 2b) (4 points)

Match the equations with the surfaces.



| Enter I,II,III,IV,V,VI here | Equation |
|-----------------------------|-------------------------|
| | $x^2 - y^2 - z^2 = 1$ |
| | $x^2 + 2y^2 = z^2$ |
| | $2x^2 + y^2 + 2z^2 = 1$ |
| | $x^2 - y^2 = 5$ |
| | $x^2 - y^2 - z = 1$ |
| | $x^2 + y^2 - z = 1$ |

Problem 3) (10 points)

a) (6 points) Find a parameterization of the line of intersection of the planes $3x - 2y + z = 7$ and $x + 2y + 3z = -3$.

b) (4 points) Find the symmetric equations

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}$$

representing that line.

Problem 4) (10 points)

a) (4 points) Find the area of the parallelogram with vertices $P = (1, 0, 0)$, $Q = (0, 2, 0)$, $R = (0, 0, 3)$ and $S = (-1, 2, 3)$.

b) (3 points) Verify that the triple scalar product has the property $[\vec{u} + \vec{v}, \vec{v} + \vec{w}, \vec{w} + \vec{u}] = 2[\vec{u}, \vec{v}, \vec{w}]$.

c) (3 points) Verify that the triple scalar product $[\vec{u}, \vec{v}, \vec{w}] = \vec{u} \cdot (\vec{v} \times \vec{w})$ has the property

$$|[\vec{u}, \vec{v}, \vec{w}]| \leq \|\vec{u}\| \cdot \|\vec{v}\| \cdot \|\vec{w}\|$$

Problem 5) (10 points)

Find the distance between the two lines

$$\vec{r}_1(t) = \langle t, 2t, -t \rangle$$

and

$$\vec{r}_2(t) = \langle 1 + t, t, t \rangle .$$

Problem 6) (10 points)

Find an equation for the plane that passes through the origin and whose normal vector is parallel to the line of intersection of the planes $2x + y + z = 4$ and $x + 3y + z = 2$.

Problem 7) (10 points)

The intersection of the two surfaces $x^2 + \frac{y^2}{2} = 1$ and $z^2 + \frac{y^2}{2} = 1$ consists of two curves.

- a) (4 points) Parameterize each curve in the form $\vec{r}(t) = (x(t), y(t), z(t))$.
- b) (3 points) Set up the integral for the arc length of one of the curves.
- c) (3 points) What is the arc length of this curve?

Problem 8) (10 points)

- a) (6 points) Find the curvature $\kappa(t)$ of the space curve $\vec{r}(t) = \langle -\cos(t), \sin(t), -2t \rangle$ at the point $\vec{r}(0)$.
- b) (4 points) Find the curvature $\kappa(t)$ of the space curve $\vec{r}(t) = \langle -\cos(5t), \sin(5t), -10t \rangle$ at the point $\vec{r}(0)$.

Hint. Use one of the two formulas for the curvature

$$\kappa(t) = \frac{|\vec{T}'(t)|}{|\vec{r}'(t)|} = \frac{|\vec{r}'(t) \times \vec{r}''(t)|}{|\vec{r}'(t)|^3},$$

where $\vec{T}(t) = \vec{r}'(t)/|\vec{r}'(t)|$. The curvatures in b) can be derived from the curvature in a). There is no need to redo the calculation, but we need a justification.

Problem 9) (10 points)

For each of the following, fill in the blank with $<$ (less than), $>$ (greater than), or $=$ (equal).

Justify your answer completely.

1. The arc length of the curve parameterized by $\vec{f}(t) = \langle \cos 2t, 0, \sin 2t \rangle$, $0 \leq t \leq \pi$. _____

The arc length of the curve parameterized by $\vec{g}(u) = \langle 3, 2 \cos u^2, 2 \sin u^2 \rangle$, $0 \leq u \leq \sqrt{\pi}$.

2. The arc length of the curve parameterized by $\vec{f}(t) = \langle t^2, 2 \cos t, 2 \sin t \rangle$, $0 \leq t \leq 2\pi$. _____

The arc length of the curve parameterized by $\vec{g}(u) = \langle u^4, 2 \cos u^2, 2 \sin u^2 \rangle$, $0 \leq u \leq 2\pi$.

3. The arc length of the curve parameterized by $\vec{f}(t) = \langle 1 + 3t^2, 2 - t^2, 5 + 2t^2 \rangle$, $0 \leq t \leq 1$. _____

The arc length of the curve parameterized by $\vec{g}(u) = \langle \frac{1}{2}u^2, u, \frac{2\sqrt{2}}{3}u^{3/2} \rangle$, $0 \leq u \leq 2$.

4. The arc length of the curve parameterized by $\vec{f}(t) = \langle \sin t, \cos t, t \rangle$, $1 \leq t \leq 5$. _____

The arc length of the curve parameterized by $\vec{g}(u) = \langle u \sin u, u \cos u, u \rangle$, $1 \leq u \leq 5$.

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| Problem 10) (10 points) |
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Given the plane $x + y + z = 6$ containing the point $P = (2, 2, 2)$. Given is also a second point $Q = (3, -2, 2)$.

a) (5 points) Find the equation $ax + by + cz = d$ for the plane through P and Q which is perpendicular to the plane $x + y + z = 6$.

b) (5 points) Find the symmetric equation for the intersection of these two planes.

