Math21A – Sug Woo Shin – 10/11

Problem: plotting points (x, y, z) satisfying an equation

Universal Recipe

(i) Substitute values $z = c_1$, $z = c_2$, ... and draw the resulting graph in the xy-plane

(ii) Bring the graphs of (i) onto the planes given by $z = c_1$, $z = c_2$, ...

(iii) Repeat (i), (ii) with (values of x, yz-plane) or (values of y, xz-plane) instead of (values of z, xy-plane).

(iv) Piece (i)-(iii) together to get the picture.

Problem: matching functions $f(x, y)$ with their graphs

Possible approaches

(i) Draw *level curves* $f(x, y) = c$ for various values of c - they should appear in the planes $z = c$. If you are given various graphs on the matching problem, slice them by the horizontal planes $z = c$ and see if their sections match the level curves you expect to see.

(ii) It is also very useful to see the *sections* by the planes $x = a$ or $y = b$ for various values of a and b.

(iii) Look at the *range of values* of $f(x, y)$. Is it always positive or negative? Can the values be arbitrarily big or arbitrarily small? Are the values always bounded below and above? For instance, note that sine and cosine functions have values between -1 and 1. Exponential functions or things like x^2, $|x|$ always have non-negative values.

(iv) How does the value of $f(x, y)$ *changes*? Does it increase/decrease in all directions or only in some directions? For example, sine and cosine functions are periodic and usually make waves.

(v) Pay attention to *symmetries* of the equation. If $f(x, y)$ is unchanged under $x \mapsto -x$ (resp. $y \mapsto -y$), it means that the graph is "mirror" symmetric with respect to the yz-plane (resp. the xz-plane). If $f(x, y)$ is unchanged under $x \leftrightarrow y$, then the graph is symmetric with respect to the plane given by $x = y$. If $f(x, y)$ is something like $x^2 + y^2$, it means that the graph is unchanged under rotations around the z-axis.

Here are exercises for drawing level curves. You don't have to be too precise - just roughly sketch them.

Exercise 1. Draw level curves of $f(x, y) = x^2 + y^2$ and $g(x, y) = x^2 - y^2$ for $z = -4, -1, 0, 1, 4$. Can you imagine the shapes of graphs of f and g?

Exercise 2. Draw level curves of $f(x, y) = \sin(x + y)$ for $z = 0, 1/2$. Draw the section sliced by the planes $x = 0$ and $x = \pi$. Can you imagine the shapes of graphs of f and g?

Exercise 3. Draw level curves of $f(x, y) = \ln(xy)$ (where $xy > 0$) for $z = 0, 1, 2$.