Math21A – 10/04 – Sug Woo Shin

Curves and surfaces in the 3-D space.

	dim	locally looks like	parametrized by	# of eqns in x,y,z
Curve	1	line	one variable	two
Surface	2	plane	two variables	one

Example 0.1. (Locally looks like...)

For a *surface,* think of a big sphere such as the earth. You are standing on the big sphere. In your eyes, it's just a huge *plane*, not a sphere. Similarly, if you walk along the equator of the earth, which is a *curve*, at each moment you would think that you are on a very long *line*, not a circle.

Example 0.2. (Number of equations in x, y, z)

We know that a sphere with center (a, b, c) and radius r is given by *one equation*

$$(x-a)^2 + (y-b)^2 + (z-c)^2 = r^2$$

We also learned that the symmetric equation of a line has the form

$$\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$$

So a line is given by *two equations* (although the equation looks like one body, observe that it breaks up into two equalities.) The basic principle is that the dimension of your object gets smaller as you give more equations to be satisfied.

Remark 0.3. (Short answer to **why parametrization?**)

We will learn many examples of curves and surfaces in this course. It is very important to *parametrize* them. First of all, it's a useful way to "write down" curves and surfaces. Once you have parametrization of curves or surfaces, "calculus"(anything related to integration and differentiation) on curves and surfaces is possible. For instance, nice formulas for arc length or surface area are available. We'll learn the arc length formula in the next class.

Some in-class exercises for section 12.1-12.2

Exercise 0.4. $\vec{r}(t) = \langle e^{t^2}, e^{-t}, t+1 \rangle$. Find the symmetric equation of the tangent line at $(e, e, 0)$.

Exercise 0.5. $\vec{r}(0) = \langle 3, 1, 2 \rangle$, $\vec{r'}(0) = \langle 0, 1, 1 \rangle$, $\vec{r''}(t) = \langle \cos t, e^t, 1 \rangle$. Find $\vec{r'}(t)$, and then $\vec{r}(t)$.

Exercise 0.6. Find all intersection points of the curve $t \mapsto \langle 1, t, t^2 \rangle$ with the plane $-5x + 2y + z + 2 = 0$.