Math 21A - Lagrange Multiplier - 11/08

Exercise 1. We are going to manufacture a cylindrical bottle which can contain exactly 3π (gallons) of water. It has metal top and bottom but cardboard side. Metal costs 1.5 times as expensive as cardboard. For what radius r and height h is the cost minimized?

Exercise 2. Find the distance between the plane $3x + 2y - 5z = 24$ and the point $(1, 1, 0)$.

(Suggestion)

Approach 1: distance formula

Approach 2: minimize the distance function

$f(x, y) = \sqrt{(x-1)^2 + (y-1)^2 + ((3x+2y-6)/5)^2}$ without constraint on x, y.

Approach 3: minimize the function $f(x, y, z)$ using the **Lagrange Multiplier** method.

Trick: the computation is easier with $f(x, y, z) = (x-1)^2 + (y-1)^2 + z^2$ than with $f(x, y, z) = \sqrt{(x-1)^2 + (y-1)^2 + z^2}$.

Exercise 3. We manufacture a rectangular box with surface area 54. What is the largest possible volume of the box?

Exercise 4. (Finding extrema on surfaces) Let $f(x, y, z) = 2x + 3y - z$. Find extrema of f on

(1) An ellipsoid $x^2 + 4y^2 + z^2 = 4$.

(2) the parametric surface $\vec{r}(u, v) = \langle u^3, v^3, 6u \rangle$.

(Suggestion)

(1) Use $g(x, y, z) = x^2 + 4y^2 + z^2 = 4$ as a constraint and apply the **Lagrange Multiplier** method.

(2) First write the function f in u,v-variables. The problem comes down to finding max/min of two variable functions *without constraint.* Find critical points and use the 2nd derivative test.