Math 21A (Sug Woo Shin) - Triple Integrals - 11/20

How to set up interated integrals for solids.

Sample problem:

Say U is the upper half ball with center (0,0,0) and radius 3. Set up the integral for $\int \int \int_U f(x, y, z) dV$.

Here is "the recipe".

1) Find the range of z in x, y: $0 \le z \le \sqrt{9 - x^2 - y^2}$.

Then collapse z-axis. (= Project onto xy-plane.)

2) Find the range of y in x: $-\sqrt{9 - x^2} \le y \le \sqrt{9 - x^2}$.

Then collapse y-axis. (= Project onto x-axis.)

3) Find the range of x: $-3 \le x \le 3$.

4) Use 1),2),3) to set up the iterated integral: $\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_{0}^{\sqrt{9-x^2-y^2}} f(x, y, z) \, dz \, dy \, dx$

Comment:

You can do this in any order of x, y, z!!

You can use the above recipe to answer the following problem.

Example 1. Evaluate $\int \int \int_U z dV$, where U is enclosed by $x = 0$, $y = 0$, $z = 0$, $x + y + z = 1$.

Example 2. Rewrite $\int_0^1 \int_{\sqrt{x}}^1 \int_0^{1-y} f(x,y,z)dzdydx$ in other integration orders.

($dxdydz$, $dxdzdy$, $dydxdz$, $dydzdx$, $dzdxdy$ are five other orders.)

Here we demonstrate: how to draw the domain of integration in 3-dim space.

(Just reverse "the recipe".)

1) Bound by $0 \leq x \leq 1$.

2) Bound by $\sqrt{x} \leq y \leq 1$. Get a region R in xy-plane.

3) Over R, bound by $z = 0$ (below) and by $z = 1 - y$ (above).

Exercise: let us rewrite with $dydxdz$.

Example 3. (revisit the volume formula) Find the volume of the solid bounded by $x^2 + y^2 = 4$, $z = x + y$, $z = -6 - x$.

Example 4. (computation of mass) $U = \{0 \leq x \leq 1, \ 0 \leq y \leq 2, \ 0 \leq z \leq xy\}$, $\delta(x,y,z) = x + 1$.

Compute $\int \int \int_U \delta(x,y,z)dV$.

(Here δ is the density function. This is the mass formula.)