

Completion of the square.

Reminder: The quadratic equation $x^2 + bx = c$ is solved by adding $(b/2)^2$ on both sides:

$$\begin{aligned}x^2 + bx &= c \\x^2 + bx + (b/2)^2 &= (b/2)^2 + c \\(x + b/2)^2 &= (b/2)^2 + c \\x + b/2 &= \sqrt{(b/2)^2 + c} \\x &= \pm\sqrt{(b/2)^2 + c} - b/2.\end{aligned}$$

The method can be used to find the center of a circle or sphere:

Example: $x^2 + 5x + y^2 - 2y = -1$ is equivalent to $(x + 5/2)^2 - 25/4 + (y - 1)^2 - 1 = -1$ or $(x - 5/2)^2 + (y - 1)^2 = (5/2)^2$. Therefore, the equation is a circle with center $(5/2, 1)$ and radius $5/2$.

Now it is your turn:

PROBLEM. Find the center and radius of the sphere
 $x^2 + 6x + y^2 - 4y + z^2 = 4 + 2z$.

1) Completion of square for x :

2) Completion of square for y :

3) Completion of square for z :

Result:

The equation is therefore a sphere of radius:

centered at the point: P=