

Mathematics 21a Fall 2006
In class problems. Oct. 26

1. Use chain rule to calculate $\frac{\partial z}{\partial t}$ and $\frac{\partial z}{\partial s}$ for $z = e^{2xy}$ and $x = \frac{s}{t}$, $y = \frac{t}{s}$.

2. A plane curve is given in polar coordinates $(r(t), \theta(t))$. Recall $x = r \cos \theta$, $y = r \sin \theta$. Calculate $\frac{dx}{dt}$, $\frac{dy}{dt}$ in terms of functions $r(t), \theta(t), r'(t), \theta'(t)$. Write down expression for the arc length element $\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}$ in the same terms.

3. Let $u = u(x, y)$, where $x = r \cos \theta$, $y = r \sin \theta$. Use chain rule to express $u_r, u_{rr}, u_{\theta\theta}$ in terms of $r, \theta, u_x, u_y, u_{xx}, u_{xy}, u_{yy}$. Show that

$$u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta} = u_{xx} + u_{yy}.$$