Mathematics 21a Fall 2006
In class problems. Dec. 7

1. Evaluate $\oint_C \overrightarrow{F} \cdot d\vec{r}$, where $\overrightarrow{F}(x, y) = \langle \arctan x + xy^2 - 7y, \sqrt{y^2 + 6} + x^2y - 2x \rangle$ and C is the circle of radius 3 in the xy-plane oriented counterclockwise.

2. Calculate $\oint\limits_C y dx - x dy$, where C is the boundary of the square R with vertices $(-1, -1)$, $(-1, 1)$, $(1, 1)$, $(1, -1)$, oriented *clockwise*.

1. Using Green's theorem in the disk D of radius 3 gives $Q_x - P_y = (2xy-2) - (2xy-7) = 5$, and hence

$$\oint_C \overrightarrow{F} \cdot d\vec{r} = \iint_D 5dA = 45\pi.$$

2. Parametrize the top edge C_t by $\langle x, 1\rangle$, $-1 \le x \le 1$. Then

$$\int_{C_t} ydx - xdy = \int_{-1}^{1} 1\, dx = 2.$$

Similarly, every other edge contributes 2 to the total integral as well. So

$$\oint_C ydx - xdy = \int_{C_t} ydx - xdy + \int_{C_r} ydx - xdy + \int_{C_b} ydx - xdy + \int_{C_l} ydx - xdy = 8.$$

Alternatively, one can use Green's theorem with $\overrightarrow{F}(x,y) = \langle y, -x\rangle$ in the square R. The clockwise orientation gives the negative sign:

$$\oint_C ydx - xdy = -\iint_R (-1-1)\, dA = 8.$$