Mathematics 21a Fall 2006
In class problems. Dec. 5

1. Show that the vector field $\overrightarrow{F} = \langle ye^{xy}, xe^{xy} - z^3, 2\sin z - 3yz^2\rangle$ is conservative in $\mathbb{R}^3$ and find a potential u with $\overrightarrow{F} = \nabla u$. Evaluate the following integral

$$\int_\gamma ye^{xy}\,dx + (xe^{xy} - z^3)\,dy + (2\sin z - 3yz^2)\,dz,$$

where γ is the straight line path in $\mathbb{R}^3$ beginning at $(0,5,0)$ and ending at $(1,0,\pi)$.

2. Let $\overrightarrow{F} = \frac{\vec{r}}{|\vec{r}|^3}$, where $\vec{r} = \langle x, y, z\rangle$ is the position vector in $\mathbb{R}^3$. Is the vector field $\overrightarrow{F}$ conservative? Try to find its potential. Is $\mathbb{R}^3 \setminus \{0\}$ simply connected?

1. The potential can be taken $u(x,y,z) = e^{xy} - yz^3 - 2\cos z$*. Then* $\overrightarrow{F} = \nabla u$*. The integral is path independent and can be evaluated by the fundumental theorem of calculus as*

$$\int_{(0,5,0)}^{(1,0,\pi)} ye^{xy}\,dx + (xe^{xy} - z^3)\,dy + (2\sin z - 3yz^2)\,dz$$
$$= u(1,0,\pi) - u(0,5,0) = (1 - 0 - 2(-1)) - (1 - 0 - 2) = 4.$$

2. $u = -\frac{1}{|\vec{r}|}$.