Mathematics 21a Fall 2006
In class problems. Nov. 16

1. Evaluate $\int_0^1 \int_{\sqrt{y}}^1 ye^{x^5}\,dx\,dy$ by changing the order of integration.

2. Calculate the area enclosed by the rose in polar coordinates $r = \cos(3\theta)$. Hint: be careful, the rose is traced over twice as θ runs form 0 to 2π.

1. Changing the order of integration we get

$$\int_0^1 \int_{\sqrt{y}}^1 ye^{x^5}\,dx\,dy = \int_0^1 \int_0^{x^2} ye^{x^5}\,dy\,dx = \int_0^1 \frac{y^2}{2}e^{x^5}\Big|_0^{x^2}\,dx = \int_0^1 \frac{1}{2}x^4 e^{x^5}\,dx = \frac{1}{10}e^{x^5}\Big|_0^1 = \frac{1}{10}(e-1).$$

2. In polar coordinates we have

$$Area = 3\int_{-\pi/6}^{\pi/6}\int_0^{\cos 3\theta} 1\,r dr\,d\theta = \frac{3}{2}\int_{-\pi/6}^{\pi/6}\cos^2 3\theta d\theta = \frac{3}{4}\int_{-\pi/6}^{\pi/6}(1-\cos 6\theta)d\theta = \frac{3}{4}\ \theta - \frac{1}{6}\sin 6\theta\Big|_{-\pi/6}^{\pi/6} = \frac{\pi}{4}.$$