

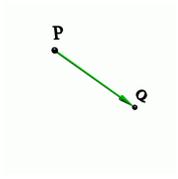
DISTANCES

Math21a, Fall 2006

DISTANCE POINT-POINT. If P and Q are two points, then

$$d(P, Q) = |\vec{PQ}|$$

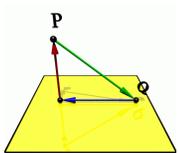
is the distance between P and Q . Unlike the book, we write $|\vec{v}| = \|\vec{v}\|$ in this handout.



DISTANCE POINT-PLANE. If P is a point in space and $\Sigma : \vec{n} \cdot \vec{x} = d$ is a plane containing a point Q , then

$$d(P, \Sigma) = \frac{|\vec{PQ} \cdot \vec{n}|}{|\vec{n}|}$$

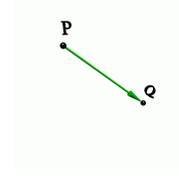
is the distance between P and the plane. Proof: use the angle formula.



EXAMPLES

DISTANCE POINT-POINT. $P = (-5, 2, 4)$ and $Q = (-2, 2, 0)$ are two points, then

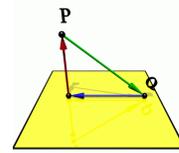
$$d(P, Q) = |\vec{PQ}| = \sqrt{(-3)^2 + 0^2 + (-4)^2} = 5$$



DISTANCE POINT-PLANE. $P = (7, 1, 4)$ is a point and $\Sigma : 2x + 4y + 5z = 9$ is a plane which contains the point $Q = (0, 1, 1)$. Then

$$d(P, \Sigma) = \frac{|(-7, 0, -3) \cdot (2, 4, 5)|}{|(2, 4, 5)|} = \frac{29}{\sqrt{45}}$$

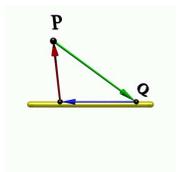
is the distance between P and Σ .



DISTANCE POINT-LINE. If P is a point in space and L is the line $\vec{r}(t) = Q + t\vec{u}$, then

$$d(P, L) = \frac{|(\vec{PQ}) \times \vec{u}|}{|\vec{u}|}$$

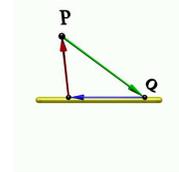
is the distance between P and the line L . Proof: the area divided by base length is height of parallelogram.



DISTANCE POINT-LINE. $P = (2, 3, 1)$ is a point in space and L is the line $\vec{r}(t) = (1, 1, 2) + t(5, 0, 1)$. Then

$$d(P, L) = \frac{|(-1, -2, 1) \times (5, 0, 1)|}{|(5, 0, 1)|} = \frac{|(-2, 6, 10)|}{\sqrt{26}} = \frac{\sqrt{140}}{\sqrt{26}}$$

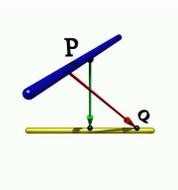
is the distance between P and L .



DISTANCE LINE-LINE. L is the line $\vec{r}(t) = Q + t\vec{u}$ and M is the line $\vec{s}(t) = P + t\vec{v}$, then

$$d(L, M) = \frac{|(\vec{PQ}) \cdot (\vec{u} \times \vec{v})|}{|\vec{u} \times \vec{v}|}$$

is the distance between the two lines L and M . Proof: the distance is the length of the vector projection of \vec{PQ} onto $\vec{u} \times \vec{v}$ which is normal to both lines.



DISTANCE LINE-LINE. L is the line $\vec{r}(t) = (2, 1, 4) + t(-1, 1, 0)$ and M is the line $\vec{s}(t) = (-1, 0, 2) + t(5, 1, 2)$. The cross product of $\langle -1, 1, 0 \rangle$ and $\langle 5, 1, 2 \rangle$ is $\langle 2, 2, -6 \rangle$. The distance between these two lines is

$$d(L, M) = \frac{|(3, 1, 2) \cdot (2, 2, -6)|}{|(2, 2, -6)|} = \frac{4}{\sqrt{44}}$$

DISTANCE PLANE-PLANE. $5x + 4y + 3z = 8$ and $5x + 4y + 3z = 1$ are two parallel planes. Their distance is

$$\frac{|8-1|}{|(5, 4, 3)|} = \frac{7}{\sqrt{50}}$$



DISTANCE PLANE-PLANE. If $\vec{n} \cdot \vec{x} = d$ and $\vec{n} \cdot \vec{x} = e$ are two parallel planes, then their distance is

$$\frac{|e-d|}{|\vec{n}|}$$

Non-parallel planes have distance 0. Proof: use the distance formula between point and plane.

