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- Please mark the box to the left which lists your section.
- Do not detach pages from this exam packet or unstaple the packet.
- Show your work! Answers without reasoning can not be given credit, except for the TF and multiple choice problems.
- Please write neatly. Answers which the grader can not read will not receive credit.
- No notes, books, calculators, computers, or other electronic aids can be used.
- All unspecified functions mentioned in this exam are assumed to be smooth: you can differentiate as many times as you want with respect to any variables.
- You have 90 minutes time to complete your work.

1		30
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		110

Problem 1) TF questions (30 points)

Mark for each of the 20 questions the correct letter. No justifications are needed.

1) T F $f(x,y)$ and $g(x,y) = f(x^2,y^2)$ have the same critical points.

2) T F If a function $f(x,y) = ax + by$ has a critical point, then $f(x,y) = 0$ for all (x,y).

3) T F $f_{xyxyx} = f_{yyxxx}$ for $f(x,y) = \sin(\cos(y + x^{14}) + \cos(x))$.

4) T F Given 2 arbitrary points in the plane, there is a function $f(x,y)$ which has these points as critical points and no other critical points.

5) T F It is possible that for some unit vector u, the directional derivative $D_u f(x,y)$ is zero even though the gradient $\nabla f(x,y)$ is nonzero.

6) T F If (x_0,y_0) is the maximum of $f(x,y)$ on the disc $x^2+y^2 \leq 1$ then $x_0^2+y_0^2 < 1$.

7) T F The linear approximation $L(x,y,z)$ of the function $f(x,y,z) = 3x+5y-7z$ at $(0,0,0)$ satisfies $L(x,y,z) = f(x,y,z)$.

8) T F If $f(x,y) = \sin(x) + \sin(y)$, then $-\sqrt{2} \leq D_u f(x,y) \leq \sqrt{2}$.

9) T F There are no functions $f(x,y)$ for which every point on the unit circle is a critical point.

10) T F An absolute maximum (x_0,y_0) of $f(x,y)$ is also an absolute maximum of $f(x,y)$ constrained to a curve $g(x,y) = c$ that goes through the point (x_0,y_0).

11) T F If $f(x,y)$ has two local maxima on the plane, then f must have a local minimum on the plane.

12) T F The acceleration of the curve $\vec{r}(t) = (\cos(t), \sin(t), t)$ at time $t = 0$ is 1.

13) T F There exists a function $f(x,y)$ of two variables which has no critical points at all.

14) T F If $f_x(x,y) = f_y(x,y) = 0$ for all (x,y) then $f(x,y) = 0$ for all (x,y) .

15) T F $(0,0)$ is a local maximum of the function $f(x,y) = x^2 - y^2 + x^4 + y^4$.

16) T F If $f(x,y)$ has a local maximum at the point $(0,0)$ with discriminant $D > 0$ then $g(x,y) = f(x,y) - x^4 + y^3$ has a local maximum at the point $(0,0)$ too.

17) T F The value of the function $f(x,y) = \sqrt{1+3x+5y}$ at $(-0.002, 0.01)$ can by linear approximation be estimated as $1 - (3/2) \cdot 0.002 + (5/2) \cdot 0.01$.

18) T F The gradient of f at a point (x_0,y_0,z_0) is tangent to the level surface of f which contains (x_0,y_0,z_0).

19) T F If $D_{\vec{v}} f(1,1) = 0$ for all vectors $\vec{v}$, then $(1,1)$ is a critical point of $f(x,y)$.

20) T F The function $u(x,t) = x^3 + t^3$ satisfies the wave equation $u_{tt} = u_{xx}$.

21) T F Every critical point (x,y) of a function $f(x,y)$ for which the discriminant D is not zero is either a local maximum or a local minimum.

22) T F The function $f(x, y) = e^y x^2 \sin(y^2)$ satisfies the partial differential equation $f_{xxyyyxyy} = 0$.

23) T F If $(0, 0)$ is a critical point of $f(x, y)$ and the discriminant D is zero but $f_{xx}(0, 0) < 0$ then $(0, 0)$ can not be a local minimum.

24) T F In the second derivative test, one can replace the condition $D > 0, f_{xx} > 0$ with $D > 0, f_{yy} > 0$ to check whether a point is a local minimum.

25) T F The gradient $\langle 2x, 2y \rangle$ is perpendicular to the surface $z = x^2 + y^2$.

26) T F If $f(x, t)$ satisfies the Laplace equation $f_{xx} + f_{tt} = 0$ and simultaneously the wave equation $f_{xx} = f_{tt}$, then $f(x, t) = ax + bt + c$.

27) T F The function $f(x, y) = (x^4 - y^4)$ has neither a local maximum nor a local minimum at $(0, 0)$.

28) T F It is possible to find a function of two variables which has no maximum and no minimum.

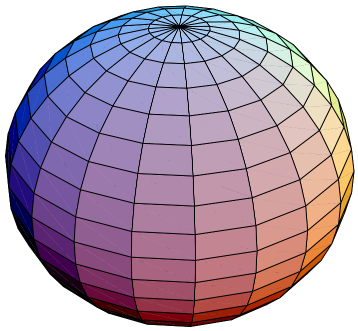
29) T F The value of the function $f(x, y) = e^x y$ at $(0.001, -0.001)$ can by linear approximation be estimated as -0.001.

30) T F For any function $f(x, y, z)$ and any unit vectors u, v, one has the identity $D_{u \times v} f(x, y, z) = D_u f(x, y, z) D_v f(x, y, z)$.

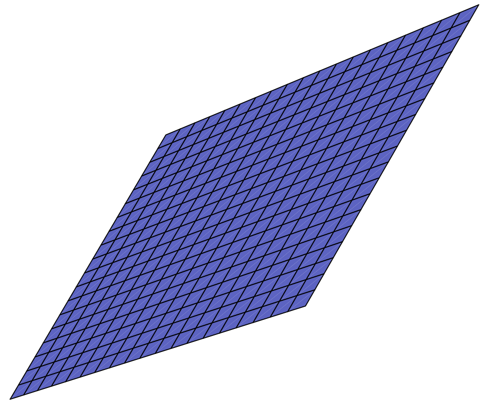
Space for work

Problem 2) (10 points)

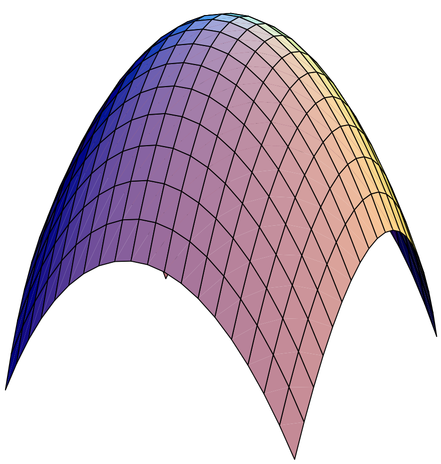
Match the parametric surfaces with their parameterization. No justification is needed.



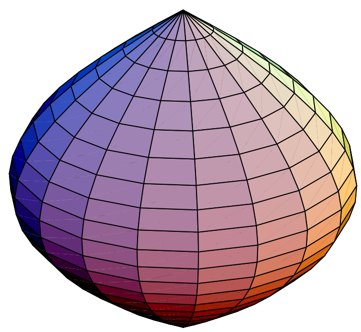
I



II



III



IV

Enter I,II,III,IV here	Parameterization
	$(u, v) \mapsto (\cos(u)\sin(v), \sin(u)\sin(v), \cos(v))$
	$(u, v) \mapsto (u - 1, v + 3, u + v)$
	$(u, v) \mapsto (u, v, 1 - u^2 - v^2)$
	$(u, v) \mapsto (\sin(v)\cos(u), \sin(v)\sin(u), v)$

Space for work

Problem 3) (10 points)

a) Show that for any differentiable function $g(x)$, the function $u(x, y) = g(x^2 + y^2)$ satisfies the partial differential equation $yu_x = xu_y$.

b) Assuming $g'(5) \neq 0$, let u be the function defined in a). Find the unit vector $\vec{v}$ in the direction of maximal increase at the point $(x, y) = (2, 1)$.

Space for work

Problem 4) (10 points)

Which point on the surface $g(x, y, z) = \frac{1}{x} + \frac{1}{y} + \frac{8}{z} = 1$ is closest to the origin?

Space for work

Problem 5) (10 points)

Find all extrema of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$ on the plane and characterize them. Do you find a absolute maximum or absolute minimum among them?

Space for work

Problem 6) (10 points)

Find all the critical points of $f(x,y) = \frac{x^5}{5} - \frac{x^2}{2} + \frac{y^3}{3} - y$ and indicate whether they are local maxima, local minima or saddle points.

Space for work

Problem 7) (10 points)

Use the technique of linear approximation to estimate $f(0.003, -0.0001, \pi/2 + 0.01)$ for

$$f(x, y, z) = \cos(xy + z) + x + 2z \ .$$

Space for work

Problem 8) (10 points)

Find the equation $ax + by + cz = d$ for the tangent plane to the level surface of

$$f(x, y, z) = \cos(xy + z) + x + 2z$$

(same function as in last problem) which contains the point $(0, 0, \pi/2)$.

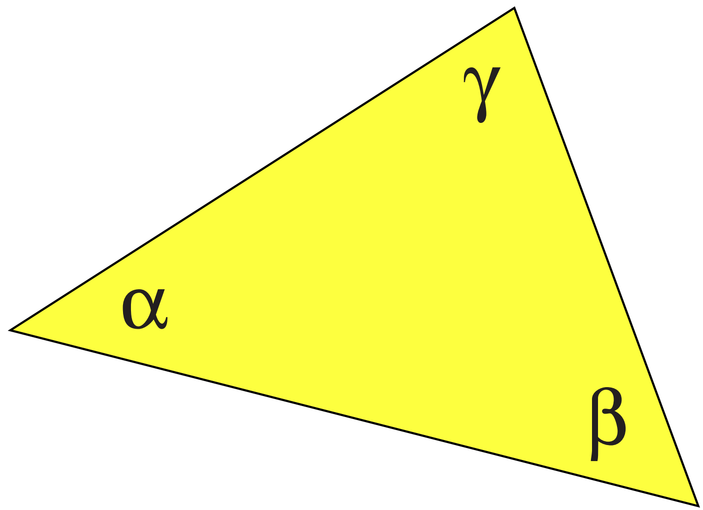
Space for work

Problem 9) (10 points)

What is the shape of the triangle with angles α, β, γ for which

$$f(\alpha, \beta, \gamma) = \log\left(\sin(\alpha)\sin(\beta)\sin(\gamma)\right)$$

is maximal?



Space for work