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- Please mark the box to the left which lists your section.
- Do not detach pages from this exam packet or unstaple the packet.
- Show your work. Answers without reasoning can not be given credit.
- Please write neatly. Answers which the grader can not read will not receive credit except for the True/False and multiple choice problems.
- No notes, books, calculators, computers, or other electronic aids can be used.
- All unspecified functions mentioned in this exam are assumed to be smooth: you can differentiate as many times as you want with respect to any variables.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		100

Problem 1) True/False questions (20 points)

Mark for each of the 20 questions the correct letter. No justifications are needed.

1) T F If $\nabla f(x, y) \neq \langle 0, 0\rangle$ at a given point (x_0, y_0), there exists a unit vector $\vec{u}$ for which $D_{\vec{u}} f(x_0, y_0)$ is zero.

2) T F If $f_{xx}(0, 0) = 0$, $D \neq 0$, and $\nabla f(0, 0) = \langle 0, 0\rangle$, then $(0, 0)$ is a saddle point.

3) T F The surface described in spherical coordinates by the equation $\rho \cos(\phi) = \rho^2 \sin^2(\phi)$ is an elliptic paraboloid.

4) T F The function $f(x, t) = x + t$ satisfies the heat equation $f_t = f_{xx}$.

5) T F $f(x, y) = 3x^2 y - y^3$ is a solution of the Laplace equation $f_{xx} + f_{yy} = 0$.

6) T F A smooth function defined on the closed unit disc $x^2+y^2 \leq 1$ has an absolute maximum in this disc (including the boundary).

7) T F A surface defined in cylindrical coordinates by the equation $g(r, \theta, z) = 0$ is always a surface of revolution.

8) T F The function $f(x, y) = x^2 - y^2$ has a neither a local maximum nor a local minimum at $(0, 0)$.

9) T F The functions $f(x, y)$ and $g(x, y) = (f(x, y))^4$ always have the same critical points.

10) T F For $f(x, y, z) = x^2 + y^2 + 2z^2$, the vector $\nabla f(1, 1, 1)$ is perpendicular to the surface $f(x, y, z) = 4$ at the point $(1, 1, 1)$.

11) T F If $f(x, y) = c$ and $f_x \neq 0$, then $\frac{dx}{dy} = f_y(x, y)/f_x(x, y)$.

12) T F $f(x, y) = \sqrt{16 - x^2 - y^2}$ has both an absolute maximum and an absolute minimum on its domain of definition.

13) T F If (x_0, y_0) is a critical point of $f(x, y)$ and $f_{xy}(x_0, y_0) < 0$, then (x_0, y_0) is a saddle point of f.

14) T F The vector $\vec{r}_v(u, v)$ of a parameterized surface $(u, v) \mapsto \vec{r}(u, v) = (x(u, v), y(u, v), z(u, v))$ is always perpendicular to the surface.

15) T F The directional derivative $D_{\vec{v}} f$ is a vector perpendicular to $\vec{v}$.

16) T F Suppose f has a maximum value at a point P relative to the constraint $g = 0$. If the Lagrange multiplier $\lambda = 0$, then P is also a critical point for f without the constraint.

17) T F At a saddle point, all directional derivatives are zero.

18) T F The minimum of $f(x, y)$ under the constraint $g(x, y) = 0$ is always the same as the maximum of $g(x, y)$ under the constraint $f(x, y) = 0$.

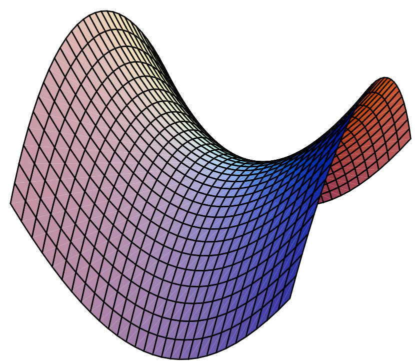
19) T F The function $f(t, x, y) = y \sin(x-t)$ satisfies the partial differential equation $f_{tt} = f_{xx} + f_{yy}$.

20) T F At a local maximum (x_0, y_0) of $f(x, y)$, one has $f_{yy}(x_0, y_0) \leq 0$.

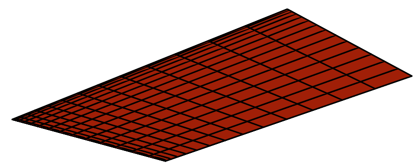
Problem 2) (10 points)

Match the parametric surfaces with their parameterization. No justifications are needed in this problem.

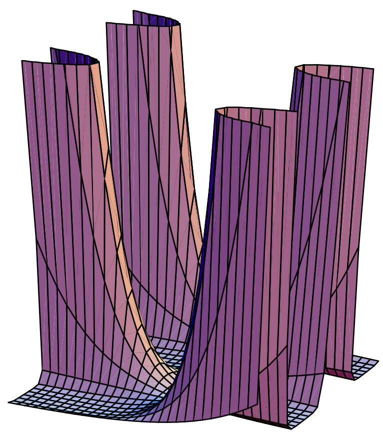
I



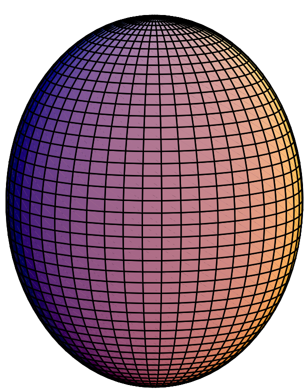
II



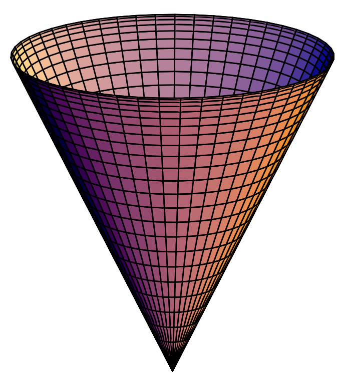
III



IV



V



Enter I,II,III,IV here	Parameterization
	$(u,v) \mapsto (\cos(u)\sin(v), \sin(u)\sin(v), 5\cos(v))$
	$(u,v) \mapsto (u^2, v^2, u^2 - v^2)$
	$(u,v) \mapsto (\cos(u)\sin(v), \sin(u)\sin(v), 5\sin(v))$
	$(u,v) \mapsto (u, v, u^2 - v^2)$
	$(u,v) \mapsto (u, v, e^{u\sin(v)})$

Problem 3) (10 points)

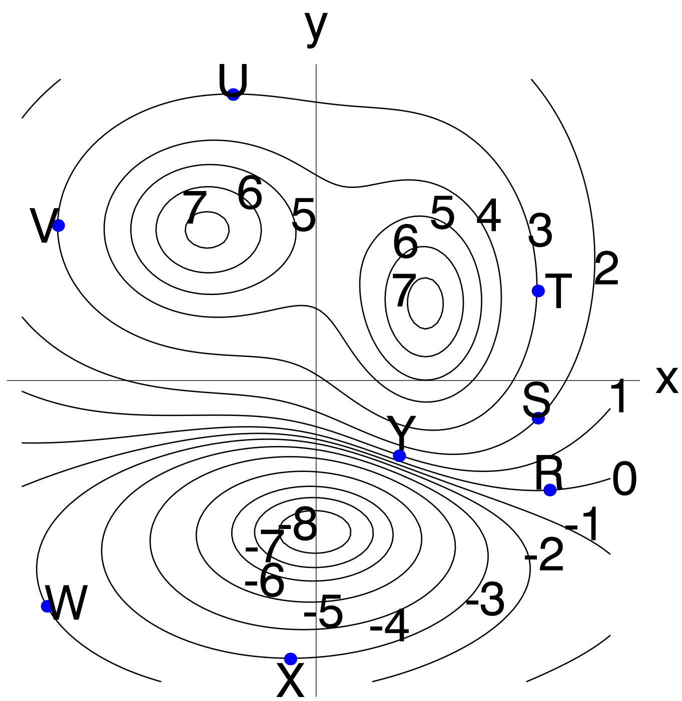
Consider the following differential equations:

A) Laplace equation $f_{xx} + f_{yy} = 0$
B) Wave equation $f_{xx} = f_{yy}$
C) Poisson equation $f_{xx} + f_{yy} = 4$
D) Heat equation $f_x = f_{yy}$
E) Transport equation $f_x = f_y$

Given the functions $g(x,y) = \sin(x+y)$ and $h(x,y) = x^2 + y^2$. Which of the partial differential equations A,B,C,D,E do they satisfy?

Equation	g is a solution	g is not a solution	h is a solution	h is not a solution
A)				
B)				
C)				
D)				
E)				

Problem 4) (10 points)



a) (4 points) Circle the point at which the magnitude of the gradient vector ∇f is greatest. Mark exactly one point. Justify your answer.

R	S	T	U	V	W	X	Y

b) (3 points) Circle the points at which the partial derivative f_x is strictly positive. Mark any number of points on this question. Justify your answers.

R	S	T	U	V	W	X	Y

c) (3 points) We know that the directional derivative in the direction $(1,1)/\sqrt{2}$ is zero at one of the following points. Which one? Mark exactly one point on this question.

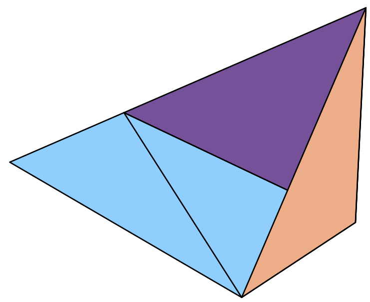
R	S	T	U	V	W	X	Y

Problem 5) (10 points)

Find all the critical points of the function $f(x,y) = \frac{x^2}{2} + \frac{3y^2}{2} - xy^3$.
For each critical point, specify if it is a local maximum, a local minimum or a saddle point and show how you know.

Problem 6) (10 points)

A beach wind protection is manufactured as follows. There is a rectangular floor $ACBD$ of length a and width b. A pole of height c is located at the corner C and perpendicular to the ground surface. The top point P of the pole forms with the corners A and C one triangle and with the corners B and C an other triangle. The total material has a fixed area of $g(a,b,c) = ab + ac/2 + bc/2 = 12$ square meters. For which dimensions a, b, c is the volume $f(a,b,c) = abc/6$ of the tetrahedral protected by this configuration maximal?



Problem 7) (10 points)

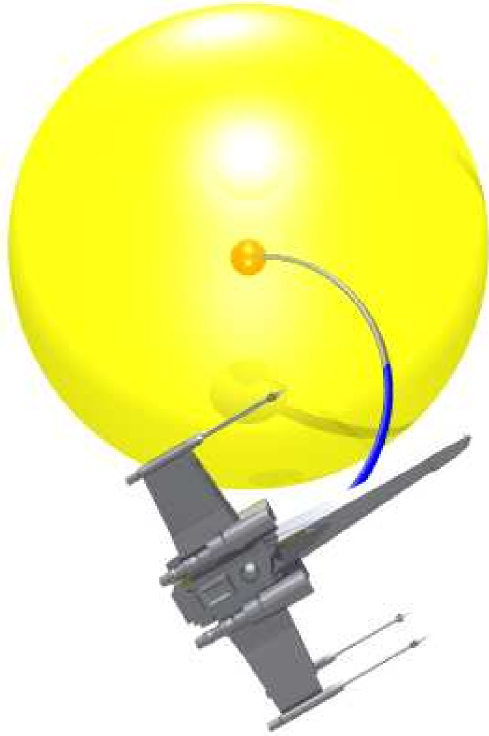
A spaceship approaches its base $B = (0, 0, -\pi/2)$ along the path

$$\mathbf{r}(t) = (\sin^2(t), 1 - \cos(t), -\pi/2 - t) .$$

The base is protected by a force shield given by the equation $x^2 + 2y^2 + z^2/\pi^2 = 3$. At time $t = -\pi/2$, the spaceship passes through the shield.

a) (5 points) At that time, does the ship pass through the shield at a right angle to the shield?

b) (5 points) The force shield is generated by a power station located at the point $(0, 0, 0)$. In the moment when the spaceship is passing through the shield, what is the rate of change of the distance from the spaceship to the power station?



Problem 8) (10 points)

Given the function
$$f(x, y) = \sqrt{105 - 2x^2 - 3y^2} \; .$$

a) (4 points) Use the technique of linear approximation at the point $(1, 1)$ to estimate $f(1.01, 0.9)$.

b) (3 points) Find a unit vector pointing in the direction at $(1, 1)$ where the function decreases fastest.

c) (3 points) Find the tangent line to the curve $\sqrt{105 - 2x^2 - 3y^2} = 10$ at the point $(1, 1)$.

Problem 9) (10 points)

Let S be the surface of revolution for which the distance r to the z-axis is $g(z) = e^z$.

a) (3 points) Find a parameterization of S.

b) (3 points) Find an implicit equation $f(x, y, z) = c$ which describes this surface.

c) (4 points) Find the tangent plane to S at the point $(-e, 0, 1)$.