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TTh11.5 Jeng-Daw Yu

- Please mark the box to the left which lists your section.
- Do not detach pages from this exam packet or unstaple the packet.
- Please write neatly. Answers which the grader can not read will not receive credit.
- No notes, books, calculators, computers, or other electronic aids can be used.
- You have 90 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
Total:		100

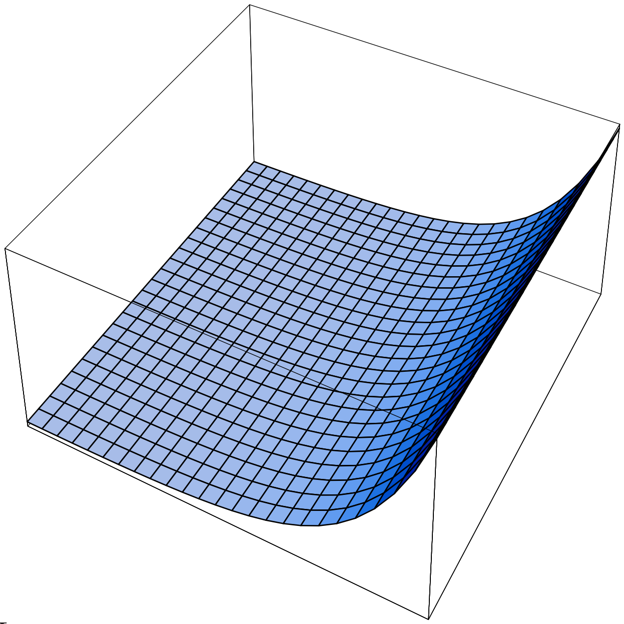
Problem 1) TF questions (20 points)

Mark for each of the 20 questions the correct letter. No justifications are needed.

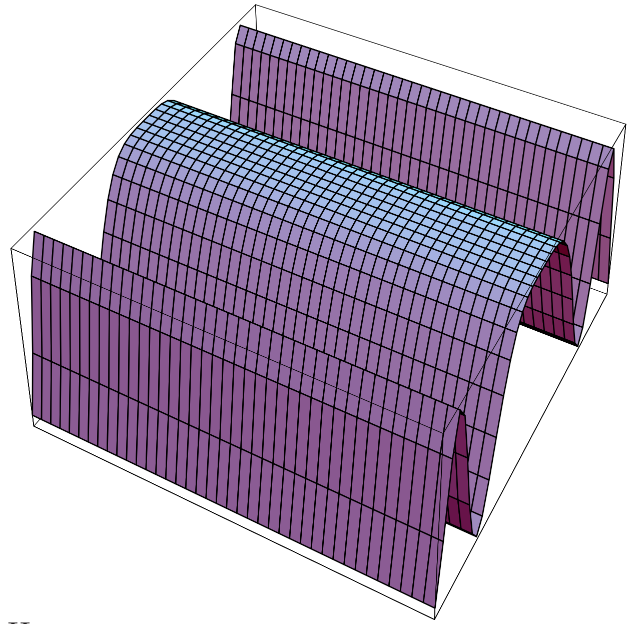
1)	T	F	The vectors $\langle 3, -2, 1\rangle$ and $\langle -6, 4, -2\rangle$ are parallel.
2)	T	F	If $\|\vec{v} \times \vec{w}\| = 0$ then $\vec{v} = \vec{0}$ or $\vec{w} = \vec{0}$.
3)	T	F	The surface $x^2 + 4y^2 = z^2 + 1$ has two sheets (that is, it consists of two surfaces which are not connected to each other).
4)	T	F	The surface $4x^2 - 4x + y^2 - 2y - 120 = -z^2$ is an ellipsoid.
5)	T	F	The parametrized lines $\vec{u}(t) = \langle 1+2t, -2-5t, 1+t\rangle$ and $\vec{v}(t) = \langle 4-4t, -12+10t, 3-2t\rangle$ are the same line.
6)	T	F	The surface $\sin(x) = z$ contains lines which are parallel to the y-axis.
7)	T	F	If $\vec{u} \cdot \vec{v} = 0$, $\vec{v} \cdot \vec{w} = 0$ and $\vec{v}$ is not the zero vector, then $\vec{u} \cdot \vec{w} = 0$.
8)	T	F	The curvature of a curve depends upon the speed at which one travels upon it.
9)	T	F	Two lines in space that do not intersect must be parallel.
10)	T	F	The intersection of the ellipsoid $x^2/3 + y^2/4 + z^2/3 = 1$ with the plane $y = 1$ is a circle.
11)	T	F	The line $\vec{r}(t) = \langle 1+2t, 1+2t, 1-4t\rangle$ hits the plane $x + y + z = 9$ at a right angle.
12)	T	F	A line in space can intersect an elliptic paraboloid in 4 points.
13)	T	F	There is a quadric which is a hyperbola when intersected with the plane $z = 0$, which is a hyperbola when intersected with the plane $y = 0$ and which is a parabola when intersected with $x = 0$.
14)	T	F	The vector $\vec{u} \times (\vec{v} \times \vec{w})$ is always in the same plane as $\vec{v}$ and $\vec{w}$.
15)	T	F	If $\vec{u} \times \vec{v} = 0$ and $\vec{u} \cdot \vec{v} = 0$, then one of the vectors $\vec{u}$ and $\vec{v}$ is zero.
16)	T	F	The triple scalar product $\vec{u} \cdot (\vec{v} \times \vec{w})$ of three vectors $\vec{u}, \vec{v}, \vec{w}$ can be a negative number.
17)	T	F	If the velocity vector $\vec{r}\,'(t)$ and the acceleration vector $\vec{r}\,''(t)$ of a curve are parallel at time $t = 1$, then the curvature $\kappa(t)$ of the curve is zero at time $t = 1$.
18)	T	F	If the speed of a parametrized curve is constant over time, then the curvature of the curve $\vec{r}(t)$ is zero.
19)	T	F	The scalar projection of a vector $\vec{v}$ onto a vector $\vec{w}$ is always equal to the scalar projection of $\vec{w}$ onto $\vec{v}$.
20)	T	F	The velocity vector of a parametric curve $\vec{r}(t)$ always has length 1.

Problem 2) (10 points)

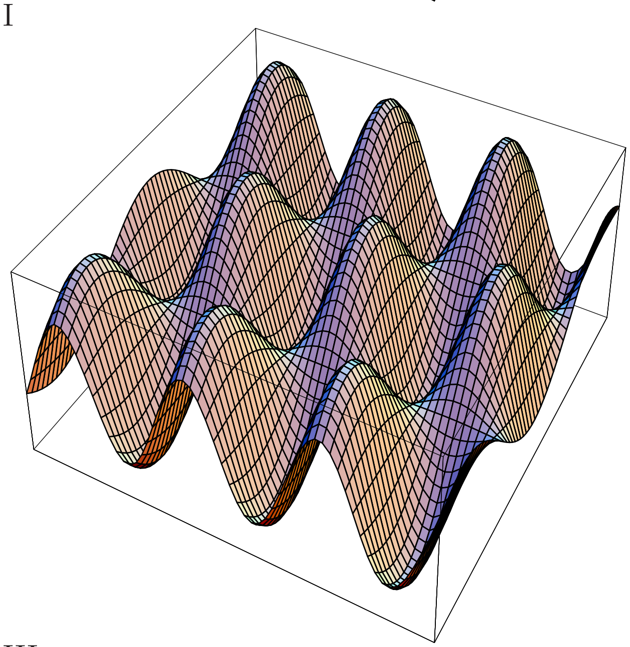
Match the equation with their graphs. No justifications are needed.



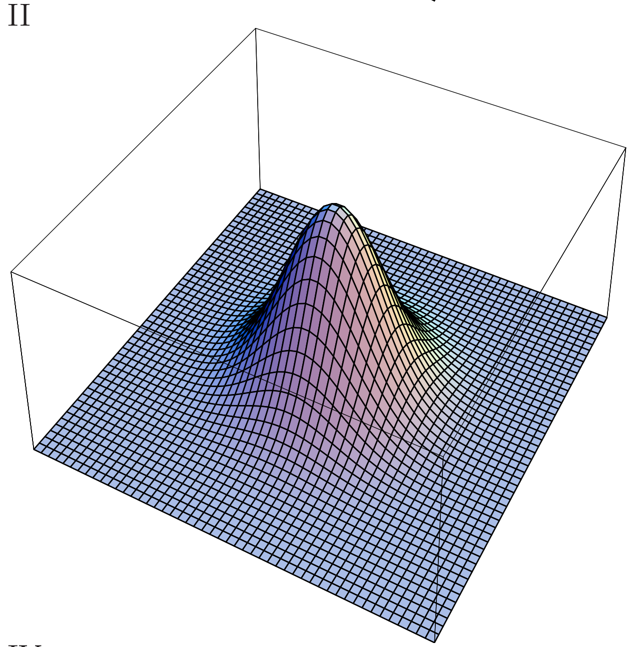
I



II



III



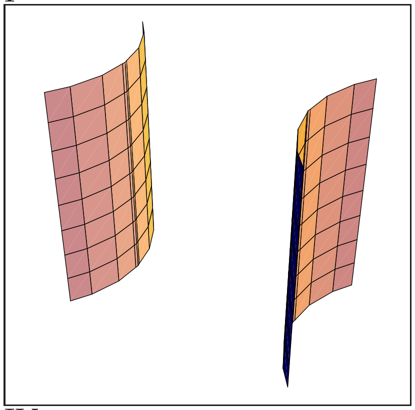
IV

Enter I,II,III,IV here	Equation
	$z = \sin(5x)\cos(2y)$
	$z = \cos(y^2)$
	$z = e^{-x^2-y^2}$
	$z = e^x$

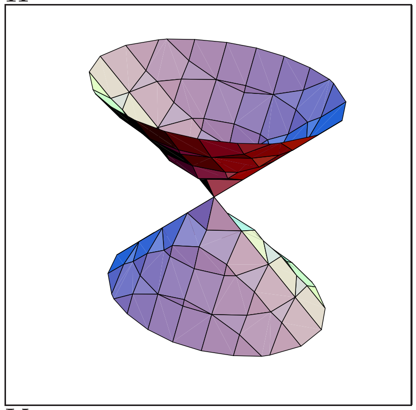
Problem 3) (10 points)

Match the equations with the surfaces.

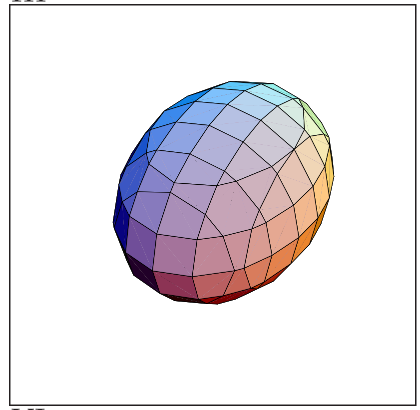
I



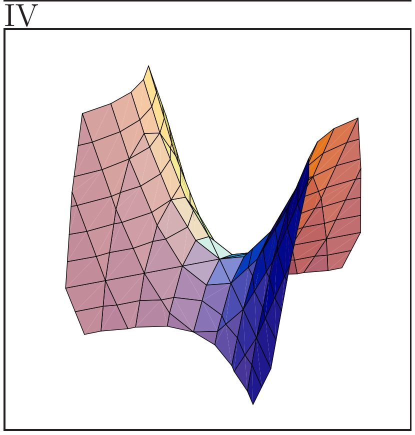
II



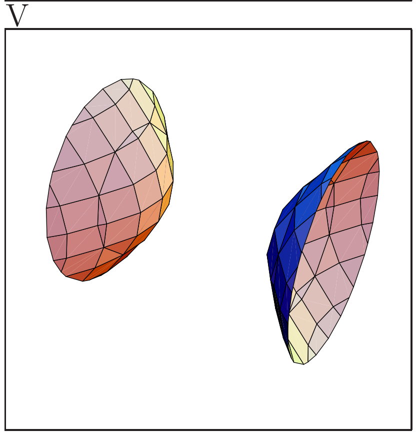
III



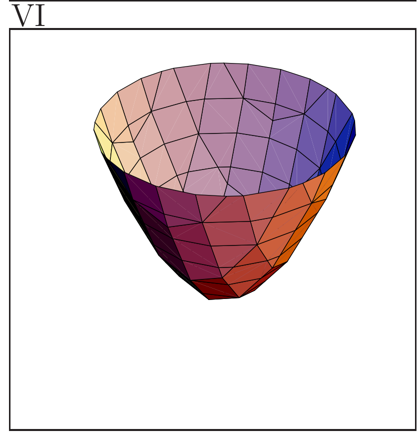
IV



V



VI



Enter I,II,III,IV,V,VI here	Equation
	$x^2 - y^2 - z^2 = 1$
	$x^2 + 2y^2 = z^2$
	$2x^2 + y^2 + 2z^2 = 1$
	$x^2 - y^2 = 5$
	$x^2 - y^2 - z = 1$
	$x^2 + y^2 - z = 1$

Problem 4) (10 points)

a) (7 points) Find a parametric equation for the line which is the intersection of the two planes $2x - y + 3z = 9$ and $x + 2y + 3z = -7$.

b) (3 points) Find a plane perpendicular to both planes and which passes through the point $P = (1, 1, 1)$.

Problem 5) (10 points)

Given the vectors $\vec{v} = \langle 1, 1, 0 \rangle$ and $\vec{w} = \langle 0, 0, 1 \rangle$ and the point $P = (2, 4, -2)$. Let Σ be the plane which goes through the origin which contains the vectors $\vec{v}$ and $\vec{w}$. Let S be the unit sphere $x^2 + y^2 + z^2 = 1$.

a) (6 points) Compute the distance from P to the plane Σ.

b) (4 points) Find the shortest distance from P to the sphere S.

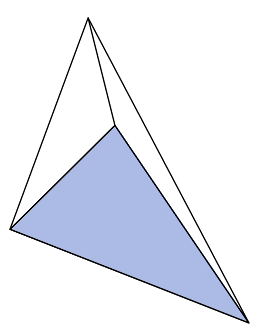
Hint for b): Find first the distance from P to the origin $O = (0, 0, 0)$.

Problem 6) (10 points)

a) (6 points) Find an equation for the plane through the points $A = (0, 1, 0), B = (1, 2, 1)$ and $C = (2, 4, 5)$.

b) (4 points) Given an additional point $P = (-1, 2, 3)$, what is the volume of the tetrahedron which has A, B, C, P among its vertices.

A useful fact which you can use without justification in b): the volume of the tetrahedron is 1/6 of the volume of the parallelepiped which has AB, AC, and AP among its edges.



Problem 7) (10 points)

The parametrized curve $\vec{u}(t) =< t, t^2, t^3 >$ (known as the "twisted cubic") intersects the parametrized line $\vec{v}(s) = \langle 1+3s, 1-s, 1+2s \rangle$ at a point P. Find the angle of intersection.

Problem 8) (10 points)

Let $\vec{r}(t)$ be the space curve $\vec{r}(t) = (\log(t), 2t, t^2)$, where $\log(t)$ is the natural logarithm (denoted by $\ln(t)$ in some textbooks).

a) What is the velocity and what is the acceleration at time $t = 1$?

b) Find the length of the curve from $t = 1$ to $t = 2$.

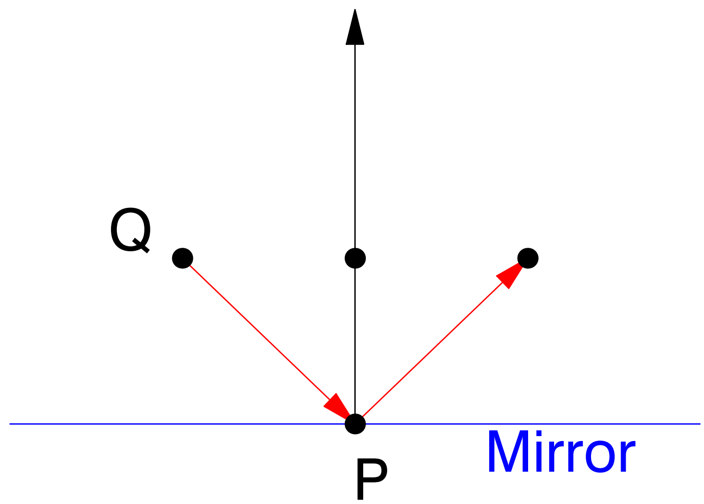
Hint: you should end up with a final integral which does not involve any square roots and which you can solve.

Problem 9) (10 points)

A planar mirror in space contains the point $P = (4, 1, 5)$ and is perpendicular to the vector $\vec{n} = \langle 1, 2, -3 \rangle$. The light ray $\vec{QP} = \vec{v} = \langle -3, 1, -2 \rangle$ with source $Q = (7, 0, 7)$ hits the mirror plane at the point P.

a) (4 points) Compute the projection $\vec{u} = \vec{\text{Proj}}_{\vec{n}}(\vec{v})$ of $\vec{v}$ onto $\vec{n}$.

b) (6 points) Identify $\vec{u}$ in the figure and use it to find a vector parallel to the reflected ray.


Q
P
Mirror