Name:

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MWF10 Oliver Knill
MWF10 Thomas Lam
MWF10 Michael Schein
MWF10 Teru Yoshida
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TTh10 Valentino Tosatti
TTh11.5 Kai-Wen Lan
TTh11.5 Jeng-Daw Yu

- Please mark the box to the left which lists your section.
- Do not detach pages from this exam packet or unstaple the packet.
- Show your work. Answers without reasoning can not be given credit except for the True/False and multiple choice problems.
- Please write neatly.
- Do not use notes, books, calculators, computers, or other electronic aids.
- Unspecified functions are assumed to be smooth and defined everywhere unless stated otherwise.
- You have 180 minutes time to complete your work.

1		20
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
11		10

12A		10
13A		10
14A		10

12B		10
13B		10
14B		10

Total:		140

Problem 1) True/False questions (20 points)

Mark for each of the 20 questions the correct letter. No justifications are needed.

1)	T	F	The length of the curve $\vec{r}(t) = \langle \sin(t), t^4 + t, \cos(t) \rangle$ on $t \in [0,1]$ is the same as the length of the curve $\vec{r}(t) = \langle \sin(t^2), t^8 + t^2, \cos(t^2) \rangle$ on $[0,1]$.
2)	T	F	The parametric surface $\vec{r}(u,v) = (5u - 3v, u - v - 1, 5u - v - 7)$ is a plane.
3)	T	F	Any function $u(x,y)$ that obeys the differential equation $u_{xx} + u_x - u_y = 1$ has no local maxima.
4)	T	F	The scalar projection of a vector $\vec{a}$ onto a vector $\vec{b}$ is the length of the vector projection of $\vec{a}$ onto $\vec{b}$.
5)	T	F	If $f(x,y)$ is a function such that $f_x - f_y = 0$ then f is conservative.
6)	T	F	$(\vec{u} \times \vec{v}) \cdot \vec{w} = (\vec{u} \times \vec{w}) \cdot \vec{v}$ for all vectors $\vec{u}, \vec{v}, \vec{w}$.
7)	T	F	The equation $\rho = \phi/4$ in spherical coordinates is half a cone.
8)	T	F	The function $f(x,y) = \begin{cases} \frac{x}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$ is continuous at every point in the plane.
9)	T	F	$\int_0^1 \int_0^x 1 \, dydx = 1/2$.
10)	T	F	Let $\vec{a}$ and $\vec{b}$ be two vectors which are perpendicular to a given plane Σ. Then $\vec{a} + \vec{b}$ is also perpendicular to Σ.
11)	T	F	If $g(x,t) = f(x - vt)$ for some function f of one variable $f(z)$ then g satisfies the differential equation $g_{tt} - v^2 g_{xx} = 0$.
12)	T	F	If $f(x,y)$ is a continuous function on $\mathbf{R}^2$ such that $\int\int_D f \, dA \geq 0$ for any region D then $f(x,y) \geq 0$ for all (x,y).
13)	T	F	Assume the two functions $f(x,y)$ and $g(x,y)$ have both the critical point $(0,0)$ which are saddle points, then $f + g$ has a saddle point at $(0,0)$.
14)	T	F	If $f(x,y)$ is a function of two variables and if $h(x,y) = f(g(y), g(x))$, then $h_x(x,y) = f_y(g(y), g(x)) g'(y)$.
15)	T	F	If we rotate a line around the z axis, we obtain a cylinder.
16)	T	F	If $u(x,y)$ satisfies the transport equation $u_x = u_y$, then the vector field $\vec{F}(x,y) = \langle u(x,y), u(x,y) \rangle$ is a gradient field.
17)	T	F	$3 \, \text{grad}(f) = \frac{d}{dt} f(x+t, y+t, z+t)$.
18)	T	F	If a vector field $\vec{F}$ is defined at all points in three-space except the origin and $\text{curl}(\vec{F}) = \vec{0}$ everywhere, then the line integral of $\vec{F}$ around any closed path not passing through the origin is zero.

TF Problems for regular sections:

19) T F If $\vec{F}$ is a vector field in space and f is equal to the line integral of $\vec{F}$ along the straight line C from $(0,0,0)$ to (x,y,z), then $\nabla f = \vec{F}$.

20) T F The line integral of $\vec{F}(x,y) = (x,y)$ along an ellipse $x^2 + 2y^2 = 1$ is zero.

21) T F The identity $\mathrm{div}(\mathrm{grad}(f)) = 0$ is always true.

TF Problem for probability theory sections:

22) T F Inside a bag is are two coins, one coin has both sides heads and one coin is normal (one head and one tail). I randomly pick one of the coins and randomly look at one side, seeing a head. Is the probability that the other side of the same coin is a tail equal to 1/2?

23) T F If X and Y are independent random variables, then $\mathrm{D}(X+Y) = \mathrm{D}(X) + \mathrm{D}(Y)$.

24) T F The expectation of the product of two random variables is always the product of the expectations.

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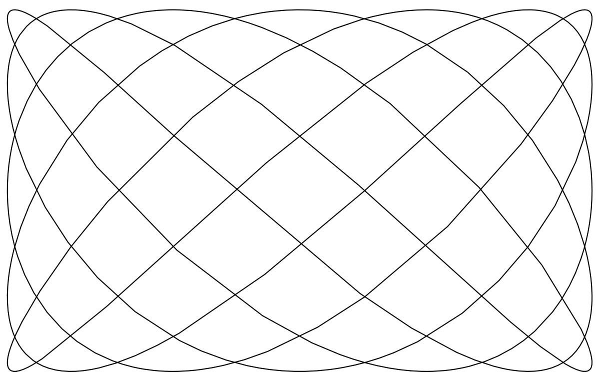
Problem 2) (10 points)

Match the equations with the curves. No justifications are needed.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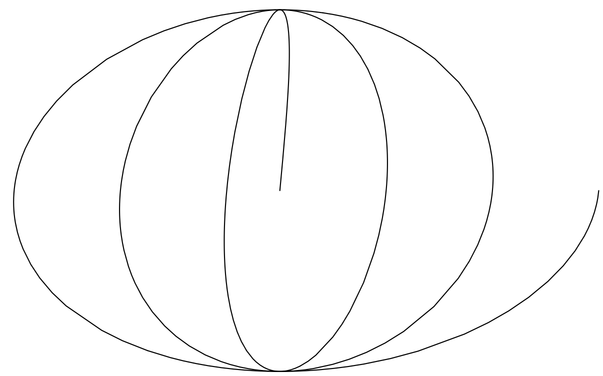
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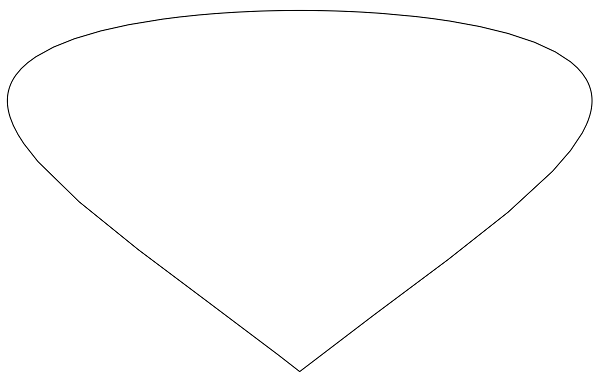
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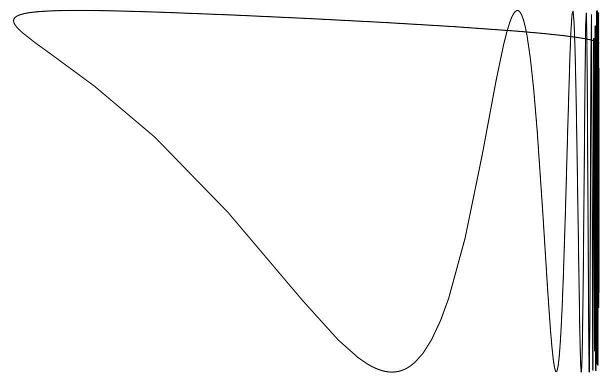

IV

Enter I,II,III,IV here	Equation
	$\vec{r}(t) = (\sin(t), t(2\pi - t))$
	$\vec{r}(t) = (\cos(5t), \sin(7t))$
	$\vec{r}(t) = (t\cos(t), \sin(t))$
	$\vec{r}(t) = (\cos(t), \sin(6/t))$

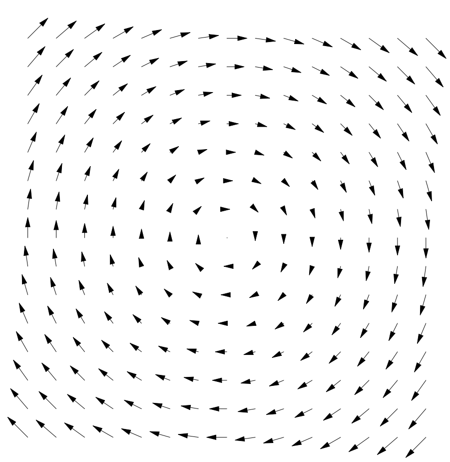
Problem 3) (10 points)

In this problem, vector fields F are written as $F = \langle P, Q \rangle$. We use abbreviations $\text{curl}(F) = Q_x - P_y$ and $\text{div}(F) = P_x + Q_y$. When stating $\text{curl}(F)(x,y) = 0$ we mean that $\text{curl}(F)(x,y) = 0$ vanishes for **all** (x,y). The statement $\text{curl}(F) \neq 0$ means that $\text{curl}(F)(x,y)$ does not vanish for at least one point (x,y). The same remark applies if curl is replaced by div.

Check the box which match the formulas of the vector fields with the corresponding picture I,II,III or IV. Mark also the places, indicating the vanishing or not vanishing of curl and div. In each of the four lines, you should finally have circled three boxes. No justifications are needed.

Vectorfield	I	II	III	IV	$\text{curl}(F) = 0$	$\text{curl}(F) \neq 0$	$\text{div}(F) = 0$	$\text{div}(F) \neq 0$
$\vec{F}(x,y) = (0,5)$								
$\vec{F}(x,y) = (y,-x)$								
$\vec{F}(x,y) = (x,y)$								
$\vec{F}(x,y) = (2,x)$								

I



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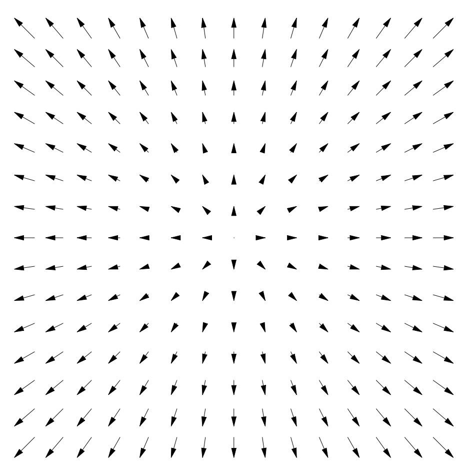

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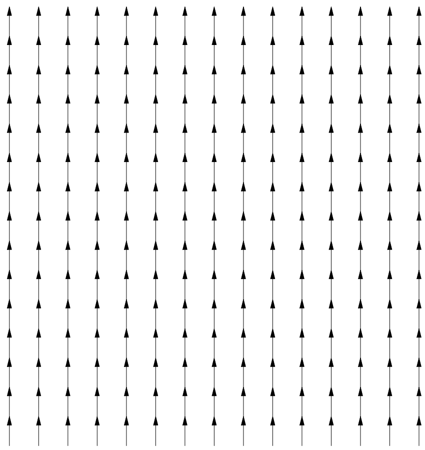
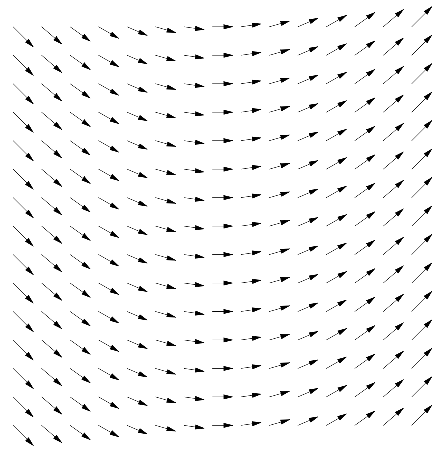




Problem 4) (10 points)

a) Find the scalar projection of the vector $\vec{v} = (3, 4, 5)$ onto the vector $\vec{w} = (2, 2, 1)$.

b) Find the equation of a plane which contains the vectors $\langle 1, 1, 0 \rangle$ and $\langle 0, 1, 1 \rangle$ and contains the point $(0, 1, 0)$.

Problem 5) (10 points)

Find the surface area of the ellipse cut from the plane $z = 2x + 2y + 1$ by the cylinder $x^2 + y^2 = 1$.

Problem 6) (10 points)

Sketch the plane curve $\vec{r}(t) = (\sin(t)e^t, \cos(t)e^t)$ for $t \in [0, 2\pi]$ and find its length.

Problem 7) (10 points)

Let $f(x, y, z) = 2x^2+3xy+2y^2+z^2$ and let R denote the region in $\mathbf{R}^3$, where $2x^2+2y^2+z^2 \leq 1$. Find the maximum and minimum values of f on the region R and list all points, where said maximum and minimum values are achieved. Distinguish between local extrema in the interior and extrema on the boundary.

Problem 8) (10 points)

Sketch the region of integration of the following iterated integral and then evaluate the integral:

$$\int_0^{\pi} \left(\int_{\sqrt{z}}^{\sqrt{\pi}} \left(\int_0^{x} \sin(xy)dy \right) dx \right) dz \ .$$

Problem 9) (10 points)

Evaluate the line integral

$$\int_C \vec{F} \cdot d\vec{r} \ ,$$

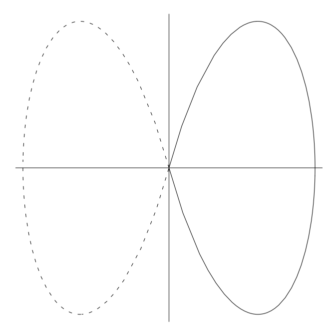
where C is the planar curve $\vec{r}(t) = (t^2, t/\sqrt{t+2}), t \in [0,2]$ and $\vec{F}$ is the vector field $\vec{F}(x,y) = (2xy, x^2 + y)$. Do this in two different ways:

a) by verifying that $\vec{F}$ is conservative and replacing the path with a different path connecting $(0,0)$ with $(4,1)$,

b) by finding a potential U satisfying $\nabla U = \vec{F}$.

Problem 10) (10 points)

Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$, where $\vec{F} = (x + e^x \sin(y), x + e^x \cos(y))$ and C is the right handed loop of the lemniscate described in polar coordinates as $r^2 = \cos(2\theta)$.



Problem 11) (10 points)

a) Find the extremal points of $f(x,y) = (x-y)^4$ on the unit circle $g(x,y) = x^2 + y^2 = 1$.
b) Find the extremal points of $g(x,y)$ subject to the constraint $f(x,y) = 4$.

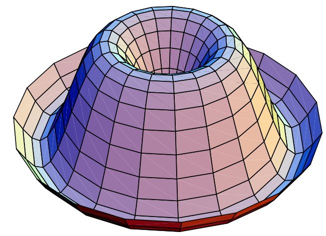
Problem 12A) (10 points)

a) Find the line integral $\int_C \vec{F} \cdot d\vec{r}$ of the vector field $\vec{F}(x,y) = (xy, x)$ along the unit circle $C : t \mapsto \vec{r}(t) = (\cos(t), \sin(t)), t \in [0, 2\pi]$ by doing the actual line integral.

b) Find the value of the line integral obtained in a) by evaluating a double integral.

Problem 13A) (10 points)

Consider the surface given by the graph of the function $z = f(x,y) = \frac{100}{1+x^2+y^2} \sin\left(\frac{\pi}{8}(x^2+y^2)\right)$ in the region $x^2 + y^2 \leq 16$. The surface is pictured to the right.



A magnetic field $\vec{B}$ is given by the curl of a vector potential $\vec{A}$. That is, $\vec{B} = \nabla \times \vec{A} = \text{curl}(A)$ and $\vec{A}$ is a vector field too. Suppose

$$\vec{A} = \left(z\sin(x^3), x\left(1-z^2\right), \log(1+e^{x+y+z})\right) .$$

Compute the flux of the magnetic field through this surface. (The surface has an upward pointing normal vector.)

Problem 14A) (10 points)

Let S be the surface given by the equations $z = x^2 - y^2$, $x^2 + y^2 \leq 4$, with the upward pointing normal. If the vector field $\vec{F}$ is given by the formula $\vec{F}(x,y,z) = \langle -x, y, \sqrt{x^2+y^2}\rangle$, find the flux of $\vec{F}$ through S.

PROBLEMS TO THE BIO-CHEM SECTIONS.

Problem 12B) (10 points)

Bob will arrive at the bus station randomly between 3:15 and 3:45 and Chuck will arrive at random between 3:00 and 4:00 (independently of Bob). Each agrees to wait up to five minutes for the other before leaving.

(a) What is the probability that they meet?
(b) Find the probability that Chuck arrives first.

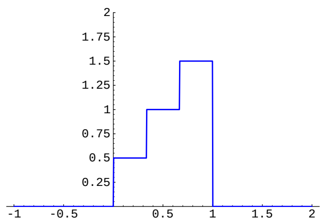
Problem 13B) (10 points)

An urn contains 10 blue balls, 8 green balls and 5 red balls. Find the probability of a blue ball being drawn before a green ball if

a) No balls are replaced after each draw.
b) If each ball is replaced after being drawn.

Problem 14B) (10 points)

Match the following density functions



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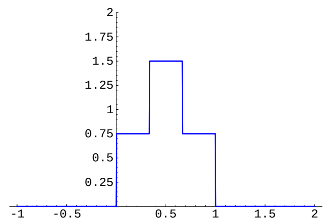

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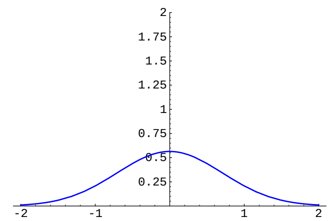

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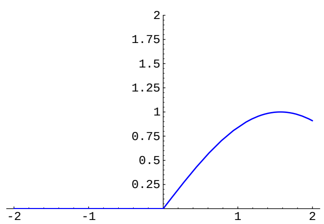

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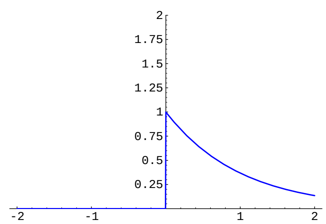

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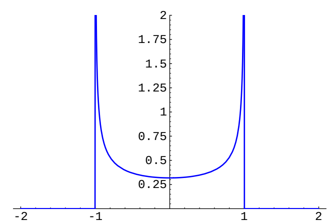
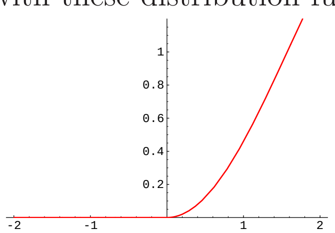
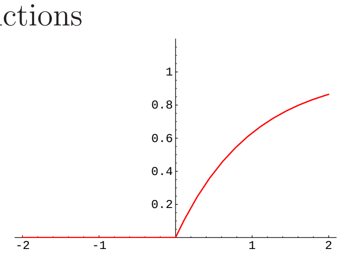


VI

with these distribution functions



a



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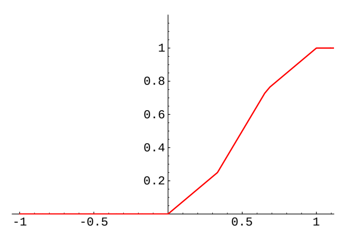

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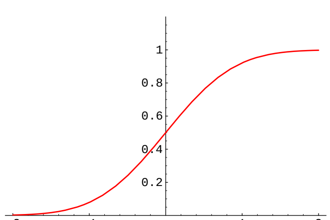

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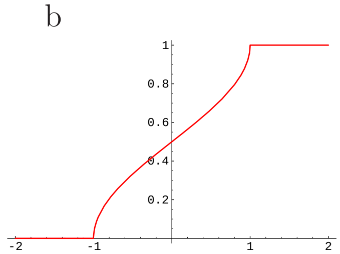

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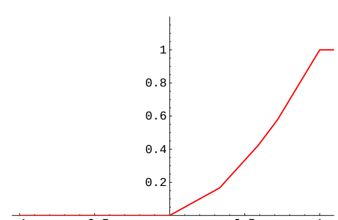

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and give reasons for your choice.