Mathematics 1a, Section 4.7 Solutions

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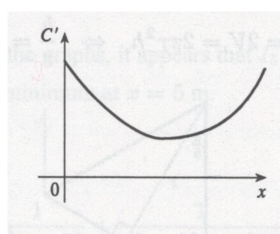
1. a. $C(0)$ represents the fixed costs of production, such as rent, utilities, machinery, etc., which are incurred even when nothing is produced.

b. The inflection point is the point at which $C''(x)$ changes from negative to positive; that is, the marginal cost $C'(x)$ changes from decreasing to increasing. Thus, the marginal cost is minimized at the inflection point.

c. The marginal cost function is $C'(x)$. We graph it as in Example 1 in Section 2.8

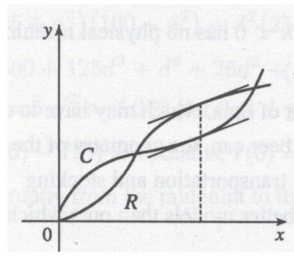


3.

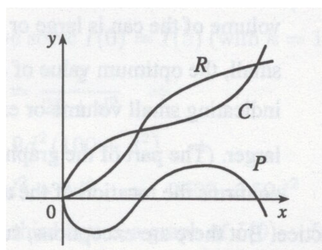
$$\begin{aligned} c(x) &= 21.4 - 0.002x = \frac{C(x)}{x} \\ C(x) &= 21.4x - 0.002x^2 \\ C'(x) &= 21.4 - 0.004x \\ C'(1000) &= 17.4 \end{aligned}$$

This means that the cost of producing the 1001st unit is about \$17.40.

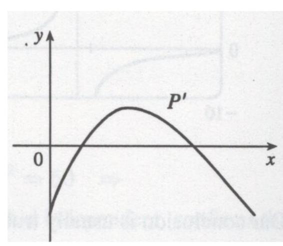
4. a. Profit is maximized when the marginal revenue is equal to the marginal cost; that is, when R and C have equal slopes. See the box preceding Example 2.



b. $P(x) = R(x) - C(x)$ is sketched.



c. The marginal profit function is defined as $P'(x)$.



5. a. The cost function is $C(x) = 40,000 + 300x + x^2$, so the cost at a production level of 1000 is $C(1000) = \$1,340,000$.. The average cost function is $c(x) = \frac{C(x)}{x} = \frac{40,000}{x} + 300 + x$ and $c(1000) = \$1340$/unit. The marginal cost function is $C'(x) = 300 + 2x$ and $C'(1000) = \$2300$/unit.

b. See the box preceding Example 1. We must have $C'(x) = c(x)$, which means $300 + 2x = \frac{40,000}{x} + 300 + x$, so $x = \sqrt{40,000} = 200$. This gives a minimum value of the average cost function $c(x)$ since $c''(x) = \frac{80,000}{x^3} > 0$.

c. The minimum average cost is $c(200) = \$700$/unit.

6. a.

$$\begin{aligned}
&C(x) = 2\sqrt{x} + \frac{x^2}{8000}\\
&C(1000) = \$188.25\\
&c(x) = \frac{C(x)}{x} = \frac{2}{\sqrt{x}} + \frac{x}{8000}\\
&c(1000) = \$0.19/\text{unit}\\
&C'(x) = \frac{1}{\sqrt{x}} + \frac{x}{4000}\\
&C'(1000) = \$0.28/\text{unit}
\end{aligned}$$

b. We must have $C'(x) = c(x)$, so

$$\begin{aligned}
&\frac{1}{\sqrt{x}} = \frac{x}{4000} = \frac{2}{\sqrt{x}} + \frac{x}{8000}\\
&\frac{x}{8000} = \frac{1}{\sqrt{x}}\\
&x^{3/2} = 8000\\
&x = 8000^{2/3} = 400
\end{aligned}$$

This is a minimum since $c''(x) = \frac{3}{2}x^{-5/2} > 0$.

c. The minimum average cost is $c(400) = \$0.15/\text{unit}$.

15. a. We are given that the demand function p is linear and $p(27,000) = 10$ and $p(33,000) = 8$, so the slope is $\frac{10-8}{27{,}000-33{,}000} = -\frac{1}{3000}$ and an equation of the line is $y - 10 = \left(-\frac{1}{3000}\right)(x - 27,000)$, so $y = p(x) = -\frac{1}{3000}x + 19$.

b. The revenue is $R(x) = xp(x) = 19x - (x^2/3000)$, so $R'(x) = 19 - (x/1500) = 0$ when $x = 28,500$. Since $R''(x) = -1/1500 < 0$, the maximum revenue occurs when $x = 28,500$, thus the price is $p(28,500) = \$9.50$.

16. a. Let $p(x)$ be the demand function. Then $p(x)$ is linear and $y = p(x)$ passes through $(20, 10)$ and $(18, 11)$, so the slope is $-\frac{1}{2}$ and an equation of the line is $y - 10 = -\frac{1}{2}(x - 20)$, or $y = -\frac{1}{2}x + 20$. Thus, the demand is $p(x) = -\frac{1}{2}x + 20$ and the revenue is $R(x) = xp(x) = -\frac{1}{x}x^2 + 20x$.

b. The cost is $C(x) = 6x$, so the profit is $P(x) = R(x) - C(x) = -\frac{1}{2}x^2 + 14x$. Then $0 = P'(x) = -x + 14$ when $x = 14$. Since $P''(x) = -1 < 0$, the selling price for maximum profit is $p(14) = -\frac{1}{2}(14) + 20 = \$13$.

18. Let x denote the number of \$10 increases in rent. Then the price is $p(x) = 800 + 10x$, and the number of units occupied is $100 - x$. Now the revenue is

$$\begin{aligned} R(x) &= \text{(price/unit)(units)} \\ &= (800 + 10x)(100 - x) = -10x^2 + 200x + 80,000 \end{aligned}$$

for $0 \le x \le 100$, this $R'(x) = -20x + 200 = 0$ when $x = 10$. This is a maximum since $R''(x) = -20 < 0$ for all x. Now we must check the value of $R(x) = (800 + 10x)(100 - x)$ at $x = 10$ and at the endpoints of the domain to see which value of x gives the maximum value of R. $R(0) = 80,000$, $R(10) = (900)(90) = 81,000$, and $R(100) = (1800)(0) = 0$. Thus, the maximum revenue of \$81,000/week occurs when 90 units are occupied at a rent of \$900/week.