Math 192r, Problem Set #9
(due 10/18/01)

1. For various small values of n (1 through 5, at least), determine the average number of times that a $2n$-step Dyck path returns to the horizontal axis (counting $(2n, 0)$ as a return but not $(0, 0)$), and conjecture a general formula. Compute at least one more value to confirm (or disprove) your conjecture. (I strongly encourage you to do this problem symbolically, if possible, using a system like Maple or Mathematica to generate and study a list whose elements are representations of the Dyck paths. Take advantage of the recursive construction of Dyck paths discussed in class, in combination with the list-manipulation operations that are available to you in these computer algebra systems.)

2. Let T_n be the number of domino tilings of a 3-by-$2n$ cylinder, obtained by gluing together the left and right sides (of length 3) of a 3-by-$2n$ rectangle.

   (a) Find a generating function for the sequence T_1,T_2,T_3,... .

   (b) Since the sequence satisfies a linear recurrence, there is a natural way to run the recurrence backward, obtaining values for T_0, T_{-1}, T_{-2}, etc. Compute T_n for all n between -10 and $+10$.

   (c) Formulate a conjecture based on your data.

   (d) Prove your conjecture.