Math 192r, Problem Set #6
(due 10/9/01)

1. For each even integer $n \geq 2$, we can represent each domino tiling of a 3-by-n rectangle by a code $(a_1, a_2, \ldots, a_n)$, where a_k is the number of vertical dominos in the kth column (always either 0 or 1). Note that two different tilings can have the same code; e.g., for $n = 2$ there are three tilings but only two codes (namely $(0, 0)$ and $(1, 1)$). Formulate a conjecture for the number of codes that occur for general n.

2. Let a_n be the number of domino tilings of a 4-by-n rectangle, with $n \geq 0$ (we put $a_0 = 1$ by convention).

   (a) Prove that the sequence $a_0, a_1, \ldots$ satisfies a linear recurrence relation of order 16 or less.

   (b) Prove that the sequence $a_0, a_1, \ldots$ satisfies a linear recurrence relation of order 8 or less.

   (c) Prove that the sequence $a_0, a_1, \ldots$ satisfies a linear recurrence relation of order 6 or less.