Math 192r, Problem Set #3
(due 9/26/01)

1. Let F_n be the nth Fibonacci number, as Wilf indexes them (with $F_0 = F_1 = 1$, $F_2 = 2$, etc.). Give a simple homogeneous linear recurrence relation satisfied by the sequence whose nth term is

   (a) nF_n;

   (b) $1F_1 + 2F_2 + ... + nF_n$;

   (c) $nF_1 + (n-1)F_2 + ... + 2F_{n-1} + F_n$;

   (d) F_n when n is odd, and 2^n when n is even.

   In each case, an explanation should be included.

2. The sequence of polynomials $f_n(x)$ in problem 2 of problem set 1 satisfies a second-order linear recurrence relation with coefficients that are Laurent polynomials in x.

   (a) Find it, and prove that it is correct. (Note that this proves your conjectures from parts (a) through (c) of that problem.)

   (b) Express $\sum_{n=0}^{\infty} f_n(x)y^n$ as a rational function of x and y.

Please be sure to write down how many hours you spent working on the assignment, and whom you worked with.