Math 192r, Problem Set #21
(due 12/13/01)

1. For $n \geq 0$ let $A(n) = \sum_{n/2 \leq k \leq n} 2^k$ (where the sum is only over integer values of k), so that $A(0) = 1$, $A(1) = 2$, $A(2) = 6$, etc. Extend $A(n)$ to the negative domain in two different ways, and check that they agree: first, by finding a formula for $A(n)$ when n is positive; and second, by applying the polytope reciprocity theorem.

2. For $n \geq 0$, let $f(n)$ be the number of integer sequences of length $n+1$ consisting of 1's, 2's, 3's, and 4's, such that the first term is 1, the last term is 1, and any two consecutive terms differ by 0 or ± 1. Thus $f(0) = 1$, $f(1) = 1$, $f(2) = 2$, $f(3) = 4$, $f(4) = 9$, etc. Show that this sequence satisfies a linear recurrence with constant coefficients, so that $f(-1), f(-2), f(-3), \ldots$ have natural values. Interpret these values combinatorially.