Math 192r, Problem Set #19
(due 12/6/01)

1. In problem #3 of assignment #17, multivariate polynomials

$$D(x_1, x_3, \ldots, x_{2n+1}; y_2, y_4, \ldots, y_{2n})$$

were defined. Find an infinite acyclic directed graph with special vertices $\ldots, v_{-1}, v_0, v_1, \ldots$ where all edges are assigned weight 1 and vertices are assigned weights according to some scheme that you must devise, so that for all integers $i \leq j$ the sum of the weights of the paths from v_i to v_j is $D(x_{2i+1}, x_{2i+3}, \ldots, x_{2j+1}; y_{2i+2}, y_{2i+4}, \ldots, y_{2j})$. Include a proof that your answer is correct.

2. Consider an infinite array with tilted upper boundary like the one shown below:

$$\begin{array}{cccccccccccccccc}
 & & & & & & & & & & & & & & & \vdots \\
 & & & & & & & & & & & & & & x_5 & \\
 & & & & & & & & & & & x_4 & & w_5 & & y_5 \\
 & & & & & & & & x_3 & & w_4 & & y_4 & & * & \\
 & & & & & x_2 & & w_3 & & y_3 & & * & & * & & * \\
 & & x_1 & & w_2 & & y_2 & & * & & * & & * & & * & \\
 & w_1 & & y_1 & & * & & * & & * & & * & & * & & * \\
\vdots & & & & & & & & & & & & & & & \vdots
\end{array}$$

Here the entries w_i, x_i, y_i are formal indeterminates, and the entries marked with asterisks are determined by the diamond rule as in assignment #17; that is, whenever the array contains four entries arranged like

$$\begin{array}{ccc} & a & \\ b & & c \\ & d & \end{array}$$

we must have $ad - bc = 1$. Some experimentation will probably convince you that each entry in the table is a Laurent polynomial in the variables w_i, x_i, y_i, and that moreover each coefficient in this polynomial equals $+1$. Show how for each such Laurent polynomial, the Laurent monomials that participate correspond to the perfect matchings of some graph (just as was the case in assignment #17). Give a concrete description of the graphs and the correspondence between matchings and monomials (including either a proof or convincingly large examples).