Math 192r, Problem Set #12
(due 11/6/01)

1. We consider directed animals on the modified square lattice that has an extra edge joining (i, j) to $(i+1, j+1)$ for all i, j. A subset S of the first quadrant is a directed animal on this lattice if for every point (i, j) in S there is a path from $(0, 0)$ to (i, j) in S via steps of the form $(+1, 0)$, $(0, +1)$, $(+1, +1)$. Let a_n be the number of directed animals on this lattice having n elements, so that $a_1 = 1$, $a_2 = 3$, $a_3 = 10$, etc. Mimic the method discussed in class for the ordinary square lattice to derive a formula for the generating function $\sum_{n=1}^{\infty} a_n$, and use this to obtain a formula for a_n itself as well as a formula for $\lim_{n\to\infty} a_n^{1/n}$.

2. (a) The mapping from the ring of formal power series to itself that sends $f(x)$ to $1 + x^2[f(x)]^3$ has a unique fixed point. Conjecture a formula for the coefficients of this formal power series. (Hint: Try to express the ratio of the coefficients of x^{2n} and x^{2n-2} as a rational function of n.)

   (b) There exist Laurent series

$$g(x) = x^{-1} - \frac{1}{2} - \frac{3}{8}x - \frac{1}{2}x^2 - \ldots$$

   and

$$g(-x) = -x^{-1} - \frac{1}{2} + \frac{3}{8}x - \frac{1}{2}x^2 + \ldots$$

   that are also fixed under that mapping. Find the first dozen coefficients of g and conjecture a formula for the coefficient of x^n.