Math 192r, Problem Set #1
(due 9/20/01)

1. (a) Write and run a program to compute $f(n) = \sum_{k=0}^{n}(-1)^k\binom{n}{k}^2$. (Submit this part by email.)
   (b) Devise a conjecture about the value of $f(n)$.
   (c) Prove your conjecture using the algebraic interpretation of $\binom{n}{k}$ as the coefficient of x^k in $(1+x)^n$.
   (d) Prove your conjecture using the interpretation of $\binom{n}{k}$ as the number of combinations of n things taken k at a time.

2. Define a sequence of functions $f_0(x), f_1(x), f_2(x), \ldots$ where $f_0(x) = x$, $f_1(x) = x$, and for all $n > 1$, $f_n(x) = ([f_{n-1}(x)]^2 + 1)/f_{n-2}(x)$. Thus, $f_2(x) = x + x^{-1}$, $f_3(x) = x + 3x^{-1} + x^{-3}$, etc.
   (a) Formulate a conjecture about the values of $f_n(1)$.
   (b) Formulate a conjecture about the values of $f_n(-1)$.
   (c) Formulate a conjecture about the values of $f_n(i)$, where $i = \sqrt{-1}$.
   (d) Formulate a conjecture about the coefficients of the polynomials $f_n(x)$.

In each case, if you can't get all the way through, explain how far you got and what the obstacles were.

Please be sure to write down how many hours you spent working on the assignment, and whom you worked with (something you should do on ALL your assignments for this course).