Math 191 Notes, 2003 September 18

1 new outline/hw
1 sectioning form
Ask for old handouts if you weren't here last time
Section: M4-5, 5-6, 8-9

Cantor's uncountable subsets

- The set of all subsets of a countably infinite set is uncountable.
  Pf: Cantor via Apolstol.
  My set $B = \{\text{all } n \text{ for which } n \notin f(n)\} = \{3, 4, \ldots\}$
  $1 : \{1, 3, \ldots\}$
  $2 : \{2, 9, \ldots\}$
  $3 : \{4, 9, \ldots\}$
  $4 :$ "all primes"
  Suppose this is element b on the list and the corresponding set is B. Suppose $b \notin B \Rightarrow b \in B$.

- Schwarzkopf attack Your job is to defend 1 mile of a border (x is from 0 to 1). Captured orders: Start at 1/2 at noon. At 12:30 flip a coin and move 1/3. At 12:45 flip a coin and move 1/9. Repeat at 1/2 the time and 1/3 the distance until 1pm, when you attack.
  Analysis: total length of intervals where N.S. won't attack.

$$\frac{1}{3} + \frac{1}{3} \cdot \frac{2}{3} + \cdots = 1$$

  Final position as a base-3 decimal $0.202022202020202\ldots$. The collection of these is known as the Cantor set (Cantor dust).

Set theory

- Complement. $A \cap B = (A^c \cup B^c)^c$. Proof: We must show that each is contained in the other. If $x \in A \cap B$, then $x \in A$, $x \notin A^c$, $x \notin B^c$. Therefore, $x \notin (A^c \cup B^c) \Rightarrow x \in (A^c \cup B^c)^c$. Now let's go backwards. If $x \in (A^c \cup B^c)^c$ then $x \notin A^c \cup B^c \Rightarrow x \notin A^c$ and $x \notin B^c \Rightarrow x \in A$ and $x \in B \Rightarrow x \in A \cap B$.

- Difference set $A \setminus B$. $A \setminus B = A \cap B^c$.

- σ-field, F. $\emptyset \in F$. If $A \in F$, $A^c \in F$. If countable collection $A_1, A_2, \ldots$ all in F, then $\bigcup_{i=1}^{\infty} A_i \in F$. (This is a countable union, because uncountable unions are weird).

Examples. $\emptyset, \Omega, \emptyset, A, A^c, \Omega,$ all subsets of A. Maybe an example? All sets of $A \in \Omega$ s.t. A is finite or A^c is finite. No! All sets of $A \in \Omega$ s.t. A is countable or A^c is countable. The union of 2 σ-fields may not be another σ-field. If the there is one subset in one of them that is not in the other and vice versa, we have a problem. The intersection is not in the union of the σ-fields.

- $\mathbb{P}$ "Probability measure" is a function $F \to [0,1]$. $\mathbb{P}(\emptyset) = 0, \mathbb{P}(\Omega) = 1$. $\mathbb{P}\left(\bigcup_{i=1}^{\infty} A_i\right) =$ sum. Where these are countable, disjoint sets.
  - $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B \setminus A) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$
  - Hint, whenever you find something with negatives, do everything with disjoint sets to be positive and then do algebra.
  - Polya, *Mathematics and Probable Inference*. How to reason through something.
  - $\mathbb{P}(A \cup B) \leq \mathbb{P}(A) + \mathbb{P}(B)$.
  - $\mathbb{P}(B \setminus A) = \mathbb{P}(B) - \mathbb{P}(A)$ if $A \subseteq B$.
  - $\mathbb{P}(A) \leq \mathbb{P}(B)$ if $A \subseteq B$.
- A non-measurable set.
  $x \sim y$ if $y = x + r, r \in \mathbb{Q}$.
  Work on $(0,1]$. Construct set H, choose 1 member from each class. Set H contains no two elements such that, $h_2 - h_1$ is rational. $(0,1]$ is a countable union of the form $H + r$.