

Math 191 Notes, 2003 September 16

Paul Bamberg Apostol introduction to probability theory – Coursepack?

SC Office hours: 2:30-3:30pm Tuesday, Science Center

Normal hours Wednesday morning, Quincy, M&W evenings, Quincy

Monte Carlo

3 Envelopes - 1 chosen at random, another (not the correct one) is revealed, switch? List permutations:

1 2 3

P H h

P h H

h P H

H P h

H h P

h H P

Functions

Given a function $f : A \rightarrow B$ (maps all elements of A into a single element of B), injective, surjective, bijective properties. If bijective, then there is the same number of elements in each set. Cardinality, trivial with finite sets, non-trivial for infinite sets.

Infinite sets

If you can establish a bijection between set of interest and natural numbers, then countably infinite. Also can be done algorithmically in a computer program that generates each item once and only once. Countable means either finite or countably infinite.

- **2-element subsets of $\{1, 2, 3, \dots\}$.** “Sum approach”, “Largest element approach”

- **Positive rational numbers.** “Diagonal” proof.

1/1 2/1 3/1 4/1

1/2 2/2 3/2 4/2

1/3 2/3 3/3 4/3

We can omit the ones that have already been listed ($1/1 = 2/2$), or we can just count them all and use the fact that it is surjective to say that this is at least countable.

- **Properties in Apostol about countability.**

- Every subset of a countable set is countable.
Pf. Set S , with subset A . Let S be x_1, x_2, x_3, \dots , circle the ones in A .
- Intersection of countable sets is countable.
Pf.
- Finite Cartesian product of countables is countable.
Pf. $P = \{1, 2, 3, \dots\}$, $P \times P = \{(1, 1), (1, 2), (2, 1), (1, 3), (2, 2), (3, 1)\}$ or you can take Cartesian product and map to $2^n 3^m$, a subset of the natural numbers, which is countable. Expand to finite number of countable sets using induction.
 $(m, n, r, \dots, s, t) \rightarrow \dots$
- Countable union of countable sets is countable.
Pf. The set $\{B_1, B_2, B_3, B_4, \dots\}$ are countable. Let the i th element in set j be $B_{i,j}$. List them in matrix form and use diagonal argument.

• Uncountable sets?

- Cantor. The set of reals satisfying $0 < x < 1$ is uncountable.
Pf. By contradiction. Assume we can make a list of all of them. Look at diagonal, replace 1 by 2, replace anything else by 1. Write this as a decimal, it cannot be on the list because it disagrees with everything in at least one place.
- Constructing a subset not in a list (extension of previous method).
For item n , flip element n to be or not be in subset.
- What is a “random real number”?
- <http://www.longbets.org/>
- Mile of border to defend. Start out at center, make a $1/3$ length step in $1/2$ time each time.