Fairness Versus Reason in the Ultimatum Game

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In the Ultimatum Game, two players are offered a chance to win a certain sum of money. All they must do is divide it. The proposer suggests how to split the sum. The responder can accept or reject the deal. If the deal is rejected, neither player gets anything. The rational solution, suggested by game theory, is for the proposer to offer the smallest possible share and for the responder to accept it. If humans play the game, however, the most frequent outcome is a fair share. In this paper, we develop an evolutionary approach to the Ultimatum Game. We show that fairness will evolve if the proposer can obtain some information on what deals the responder has accepted in the past. Hence, the evolution of fairness, similarly to the evolution of cooperation, is linked to reputation.

The Ultimatum Game is quickly catching up with the Prisoner's Dilemma as a prime showpiece of apparently irrational behavior. In the past two decades, it has inspired dozens of theoretical and experimental investigations. The rules of the game are surprisingly simple. Two players have to agree on how to split a sum of money. The proposer makes an offer. If the responder accepts, the deal goes ahead. If the responder rejects, neither player gets anything. In both cases, the game is over. Obviously, rational responders should accept even the smallest positive offer, since the alternative is getting nothing. Proposers, therefore, should be able to claim almost the entire sum. In a large number of human studies, however, conducted with different incentives in different countries, the majority of proposers offer 40 to 50% of the total sum, and about half of all responders reject offers below 30% (1–6).

The irrational human emphasis on a fair division suggests that players have preferences which do not depend solely on their own payoff, and that responders are ready to punish proposers offering only a small share by rejecting the deal (which costs less to themselves than to the proposers). But how do these preferences come about? One possible explanation is that the players do not grasp that they interact only once. Humans are accustomed to repeated interactions. Repeating the Ultimatum Game is like haggling over a price, and fair splits are more likely (6–8). Another argument is based on the view that allowing a co-player to get a large share is conceding a relative advantage to a direct rival. This argument holds only for very

small groups, however: a simple calculation shows that responders should only reject offers that are less than $1/n$th of the total sum, where n is the number of individuals in the group (9). A third explanation is based on the idea that a substantial proportion of humans maximize a subjective utility function different from the payoff (10–12).

Here, we studied the Ultimatum Game from the perspective of evolutionary game theory (13). To discuss this model, both analytically and by means of computer simulations, we set the sum which is to be divided equal to 1 and assumed that players are equally likely to be in either of the two roles. Their strategies are given by two parameters p and $q \in [0,1]$. When acting as proposer, the player offers the amount p. When acting as responder, the player rejects any offer smaller than q. The parameter q can be seen as an aspiration level. It is reasonable to assume that the share kept by the player acting as proposer, $1 - p$, should not be smaller than the aspiration level, q. Therefore, only strategies with $p + q \leq 1$ were considered (14).

The expected payoff for a player using strategy $S_1 = (p_1, q_1)$ against a player using $S_2 = (p_2, q_2)$ is given (up to the factor 1/2, which we henceforth omit) by (i) $1 - p_1 + p_2$, if $p_1 \geq q_2$ and $p_2 \geq q_1$; (ii) $1 - p_1$, if $p_1 \geq q_2$ and $p_2 < q_1$; (iii) p_2, if $p_1 < q_2$ and $p_2 \geq q_1$; and (iv) 0, if $p_1 < q_2$ and $p_2 < q_1$.

Before studying the full game, with its continuum of strategies, let us first consider a so-called minigame with only two possible offers h and l (high and low), with $0 < l < h < 1/2$ (9, 15). There are four different strategies, (l,l), (h,l), (h,h), and (l,h), which we enumerate, in this order, by G_1 to G_4. G_1 is the "reasonable" strategy of offering little and rejecting nothing [for the cognoscenti: it is the only subgame perfect Nash equilibrium of the minigame (16)]. G_2 makes a high offer but is willing to accept a low offer. G_3 is the "fair" strategy, offering and demanding a high share. For the sake of exposition, we

omit G_4, which gets eliminated anyway. To describe the change in the frequencies x_1, x_2, and x_3 of the strategies G_1, G_2, and G_3, respectively, we use the replicator equation. It describes a population dynamics where successful strategies spread, either by cultural imitation or biological reproduction (17). Under these dynamics, the reasonable strategy G_1 will eventually reach fixation. Populations that consist only of G_1 and G_3 players will converge to pure G_1 or G_3 populations depending on the initial frequencies of the two strategies. Mixtures of G_1 and G_2 players will always tend to G_1, but mixtures of G_2 and G_3 players are neutrally stable and subject to random drift. Hence, starting with any mixture of G_1, G_2, and G_3 players, evolution will always lead to a population that consists entirely of G_1 players (18). Reason dominates fairness.

Let us now introduce the possibility that players can obtain information about previous encounters. In this case, individuals have to be careful about their reputation: if they accept low offers, this may become known, and the next proposer may think twice about making a high offer. Assume, therefore, that the average offer of an h-proposer to an l-responder is lowered by an amount a. Even if this amount is very small—possibly because obtaining information on the co-player is difficult or because the information may be considered unreliable by h-proposers—the effect is drastic (19). In a mixture of h-proposers only, the fair strategy, G_3 dominates. The whole system is now bistable: depending on the initial condition, either the reasonable strategy G_1 or the fair strategy G_3 reaches fixation (Fig. 1). In the extreme case, where h-proposers have full information on the responder's type and offer only l when they can get away with it, we observe a reversal of the game: G_3 reaches fixation whereas mixtures between G_1 and G_2 are neutrally stable. Intuitively, this reversal occurs because it is now the responder who has the initiative: it is up to the proposer to react.

For $0 < a < h - l$, G_3 risk-dominates (20): this implies that whenever stochastic fluctuations are added to the population (by allowing mutation, for instance, or spatial diffusion), the fair strategy will supersede the reasonable one in the long run (Fig. 1).

Let us now study the evolutionary dynamics on the continuum of all strategies, $S(p,q)$. Consider a population of n players. In every generation, several random pairs are formed. Suppose each player will be proposer on average r times and be responder the same number of times. The payoffs of all individuals are then summed up. For the next generation, individuals leave a number of offspring proportional to

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their total payoff. Offspring adopt the strategy of their parents, plus or minus some small random value. Thus, this system includes selection and mutation. As before, we can interpret these dynamics as denoting biological or cultural reproduction. We observe that the evolutionary dynamics lead to a state where all players adopt strategies that are close to the rational strategy, $S(0,0)$.

Let us now add the possibility that a proposer can sometimes obtain information on what offers have been accepted by the responder in the past. We stress that the same players need not meet twice. We assume that a proposer will offer, whatever is smaller, his own p-value or the minimum offer that he knows has been accepted by the responder during previous encounters. In addition, we include a small probability that proposers will make offers that are reduced by a small, randomly chosen amount. This effect allows a proposer to test for responders who are willing to accept low offers. Hence, p can be seen as a proposer's maximum offer, whereas q represents a responder's minimum acceptance

level. Each accepted deal is made known to a fraction w of all players. Thus, individuals who accept low offers run the risk of receiving reduced offers in the future. In contrast, the costly act of rejecting a low offer buys the reputation that one accepts only fair offers. Figure 2 shows that this process can readily lead to the evolution of fairness. The average p and q values depend on the number of games per individual, r, and the fraction w of individuals who find out about any given interaction. Larger r and w values lead to fairer solutions.

Hence, evolutionary dynamics, in accordance with the predictions of economic game theory, lead to rational solutions in the basic Ultimatum Game. Thus, one need not assume that the players are rational utility-maximizers to predict the prevalence of low offers and low aspiration levels. Whatever the evolutionary mechanism—learning by trial and error, imitation, inheritance—it always promotes the same reasonable outcome: low offers and low demands.

If, however, we include the possibility that individuals can obtain some information on which offers have been accepted by

others in previous encounters, the outcome is dramatically different. Under these circumstances, evolutionary dynamics tend to favor strategies that demand and offer a fair share of the prize. This effect, which does not require the same players to interact twice, suffices to keep the aspiration levels high. Accepting low offers damages the individual's reputation within the group and increases the chance of receiving reduced offers in subsequent encounters. Rejecting low offers is costly, but the cost is offset by gaining the reputation of somebody who insists on a fair offer. When reputation is included in the Ultimatum Game, adaptation favors fairness over reason. In this most elementary game, information on the co-player fosters the emergence of strategies that are nonrational, but promote economic exchange. This agrees well with findings on the emergence of cooperation (21) or of bargaining behavior (22). Reputation based on commitment and communication plays an essential role in the natural history of economic life (23).

Fig. 1. Fairness dominates in the mini-ultimatum game if proposers have some chance of finding out whether responders might accept a low offer. There are three strategies: the reasonable strategy $G_1(l,l)$ offers and accepts low shares; the fair strategy $G_3(h,h)$ offers and accepts high shares; the strategy $G_2(h,l)$ offers high shares but is willing to accept low shares. If there is no information on the responder's type, $a = 0$, then the reasonable strategy G_1 dominates the overall dynamics: G_1 and G_3 are bistable, G_2 and G_3 are neutral, but G_1 dominates G_2. If there is some possibility of obtaining information on the responder's type, then we assume that h-proposers will reduce their average offers to l-responders by an amount a. For $0 < a < h - l$, both G_1 and G_3 dominate G_2. G_1 and G_3 are still bistable, but the fair strategy has the larger basin of attraction; adding noise or spatial effects will favor fairness. In the special limit, $a = h - l$, which can be interpreted as having full information on the responders type, the game is reversed: G_1 and G_2 are neutral, whereas G_3 dominates G_2; G_3 is the only strict Nash solution. The figure shows the flow of evolutionary game dynamics (17) on the edge of the simplex S_3 (18, 19).

Reason   Fairness

G_2

$G_1 \longleftrightarrow G_3$

$a = 0$

G_2

$G_1 \longleftrightarrow G_3$

$0 < a < h - 1$

G_2

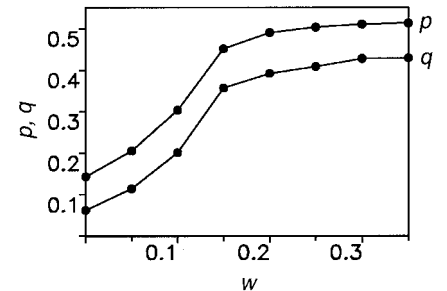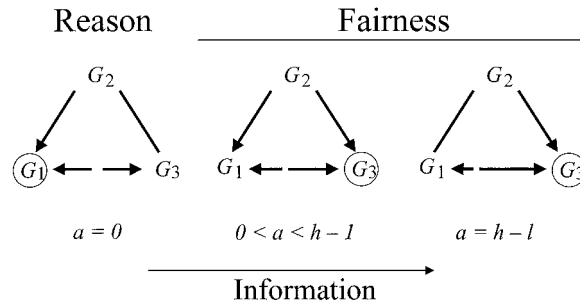$G_1 \longleftrightarrow G_3$

$a = h - l$

Information

Fig. 2. Fairness evolves in computer simulations of the Ultimatum Game, if a sufficiently large fraction w of players is informed about any one accepted offer. Each player is defined by an $S(p,q)$ strategy with $p + q \leq 1$ (14). In any one interaction, a random pair of players is chosen. The proposer will offer—whatever is smaller—either his own p value or the lowest amount that he knows was accepted by the responder during previous interactions. In addition, there is a small (0.1) probability that the responder will offer his p value minus some random number between 0 and 0.1; this is to test for players who are willing to accept reduced offers. The total population size is $n = 100$. Individuals reproduce proportional to their payoff. Offspring adopt their parent's p and q values plus a random number from the interval (-0.005, $+0.005$). There are on average $r = 50$ rounds per player per generation in both roles. Equilibrium p and q values are shown averaged over 10^5 generations. For $w = 0$ (no information about previous interactions), the p and q values converge close to the rational solution $S(0,0)$; they are not exactly zero because mutation introduces heterogeneity, and the best response to a heterogeneous population is not $S(0,0)$. For increasing values of w, there is convergence close to the fair solution, $S(1/2,1/2)$, with q being slightly smaller than p.

Table 1. Payoff matrix for the mini-ultimatum game.

	G_1	G_2	G_3	G_4
G_1	1	$1 - l + h$	h	l
G_2	$1 - h + l$	1	1	$1 - h + l$
G_3	$1 - h$	1	1	$1 - h$
G_4	$1 - l$	$1 - l + h$	h	0

Table 2. Payoff matrix for the mini-ultimatum game with information.

	G_1	G_2	G_3	G_4
G_1	1	$1 - l + h - a$	$h - a$	l
G_2	$1 - h + l + a$	1	$1 - a$	$1 - h + l$
G_3	$1 - h + a$	$1 + a$	1	$1 - h$
G_4	$1 - l$	$1 - l + h$	h	0

References and Notes

1. W. Güth, R. Schmittberger, B. Schwarze, *J. Econ. Behav. Organ.* **3**, 376 (1982).
2. R. H. Thaler, *J. Econ. Perspect.* **2**, 195 (1988).
3. W. Güth and R. Tietze, *J. Econ. Psychol.* **11**, 417 (1990).
4. E. Fehr and S. Gächter, *Homo Reciprocans and Human Cooperation*, discussion paper, Institute for Empirical Economic Research, University of Zurich (1999); E. Fehr and K. Schmidt, *Q. J. Econ.* **114**, 817 (1999).
5. G. E. Bolton and R. Zwick, *Game Econ. Behav.* **10**, 95 (1995).
6. A. E. Roth, in *Handbook of Experimental Economics*, J. H. Kagel and A. E. Roth, Eds. (Princeton Univ. Press, Princeton, NJ, 1995); ———, V. Prasknikar, M. Okuno-Fujiwara, S. Zamir, *Am. Econ. Rev.* **81**, 1068 (1991).
7. A. Rubinstein, *Econometrica* **50**, 97 (1982).
8. G. E. Bolton and A. Ockenfels, *Am. Econ. Rev.* **90**, 166 (2000).
9. S. Huck and J. Öchssler, *Game Econ. Behav.* **28**, 13 (1999).
10. G. Kirchsteiger, *J. Econ. Behav. Organ.* **25**, 373 (1994).
11. J. Bethwaite and P. Tompkinson, *J. Econ. Psychol.* **17**, 259 (1996).
12. D. Kahnemann, J. L. Knetsch, R. Thaler, *J. Bus.* **59**, 5285 (1986); E. Fehr and K. M. Schmidt, *Q. J. Econ.* **114**, 817 (1999).
13. J. Maynard Smith, *Evolution and the Theory of Games* (Cambridge Univ. Press, Cambridge, 1982).
14. The condition, $1 - p \geq q$, is equivalent to the assumption that the individuals do not regard the role of proposer as inferior to the role of responder. Therefore, what they demand for themselves when acting as proposer should not be less than the minimum amount they expect as responders. This condition is relevant when introducing information into the Ultimatum Game. Otherwise, high levels of information can lead to a reversal of the game. If the proposer has perfect knowledge of the responder's q value, then it is in fact the responder who makes the offer. In this case, evolutionary dynamics lead to strategies close to $S(1,1)$; that is, proposers have to offer almost the full amount. The condition $1 - p \geq q$ avoids this, perhaps unrealistic, complication.
15. J. Gale, K. Binmore, L. Samuelson, *Game Econ. Behav.* **8**, 56 (1995).
16. K. G. Binmore, *Fun and Games: A Text on Game Theory* (Heath, Lexington, MA, 1992).
17. J. Hofbauer and K. Sigmund, *Evolutionary Games and Population Dynamics* (Cambridge Univ. Press, Cambridge, 1998).
18. In the minigame, we consider the strategies $G_1(l,l)$, $G_2(h,l)$, $G_3(h,h)$, and $G_4(l,h)$. The matrix of the expected payoff values is shown in Table 1. We note first that $G_4(l,h)$ is dominated by $G_1(l,l)$ so that x_4/x_1 converges to 0. It follows that all orbits in the interior of the state space S_4 (the simplex spanned by the unit vectors on the x_i axes) converge to the boundary face, where $x_4 = 0$. The edge G_2G_3 (high proposers only) consists of fixed points. Those fixed points between G_3 and $Q(0,[1 - h]/[1 - l], [h - l]/[1 - l],0)$ are saturated. They cannot be invaded by low proposers and correspond to Nash equilibria. The other points can be invaded by low proposers, G_1 and G_4. On the edge G_1G_2, strategy G_1 dominates. The edge G_1G_3 is bistable, with fixed point $P(1 - h,0,h,0)$. There is one further fixed point, $R(0,1 - h + l,0,h - l)$ which is stable on the edge G_2G_4 but can be invaded by both missing strategies. The dynamics are simple: orbits can converge either to G_1, which is the only perfect Nash equilibrium of the game, or else to the segment QG_3. But once there, neutral drift or recurrent mutations introducing G_1 or G_4 will inexorably push the state toward QG_2 (it is enough to note that x_2/x_3 grows whenever x_1 or x_4 are positive) so that eventually the reasonable strategy G_1 reaches fixation.
19. Let us now assume that if a player has a low aspiration level, then this risks to become known to the proposer, in which case an h-proposer will, with a certain probability, offer a bit less. Suppose the average offer of an h-proposer to an l-responder is $h - a$, which is $\geq l$. This implies that the payoff matrix is now given by Table 2. For $0 < a < h - l$, the fair strategy G_3 dominates G_2 on the edge G_2G_3. On the edge G_1G_3, the system is bistable: the basin of attraction of G_1 and G_3 are separated by the point $P(a) = (1 - h + a, 0, h - a, 0)$. The game dynamics have the following properties: (i) The ratio x_2x_4/x_1x_3 remains constant. (ii) There exists a line of fixed points in the interior S_4, of the form $x_i = m_i + \mu$ (for $i = 1,3$) and $x_i = m_i - \mu$ (for $i = 2,4$) where μ is a real parameter and $(m_1,m_2,m_3,m_4) = (1/S)[a(1 - h), l(1 - h), l(h - l - a), a(h - l-a)]$ with $S = (l + a)(1 - l - a)$. (iii) This line intersects all invariant surfaces of the form $x_2x_4 = Kx_1x_3$ (for $K > 0$) in a fixed point which is a saddle on this surface (the surface is spanned by the edges $G_1G_2G_3G_4G_1$). (iv). The corresponding stable manifolds divide the state space S_4 into two regions, a basin of attraction for G_1 and a basin of attraction for G_3. (v) The line intersects either the face $x_4 = 0$ or $x_2 = 0$; the intersection with $x_4 = 0$ is a saddle point $Q(a)$ on this face. There is no other fixed point on this face. If a varies from 0 to $h - l$, $Q(a)$ describes an arc from the point Q to the point $S(1 - l/h, l/h, 0,0)$. For some values of h and l, this arc will cross the edge G_1G_3; at such an intersection, $Q(a) = P(a)$. (vi) If $a = h - l$ (total knowledge of the co-player, full exploitation of his weakness) we get a phase portrait on the $G_1G_2G_3$-simplex which is exactly the reverse of the phase portrait if $a = 0$: the Nash equilibria are G_3 and the points on the segment G_1S. (vii) Finally, we note that on the G_1G_3 edge, $P(a)$ separates the basins of attraction: if initially x_1 is larger than $1 - h + a$, then G_1 reaches fixation. Thus, G_3 has the larger basin of attraction (it is risk-dominant) whenever $h < 1/2 + a$.
20. H. P. Young, *Econometrica* **61**, 57 (1993); M. Kandori, G. J. Mailath, R. Rob, *Econometrica* **61**, 29 (1993); J. Hofbauer, in *Game Theory, Experience, Rationality: Foundations of Social Sciences, Economics and Ethics*, W. Leinfellner and E. Köhler, Eds. (Kluwer, Dordrecht, Netherlands, 1998), pp. 245–267.
21. M. A. Nowak and K. Sigmund, *Nature* **393**, 573 (1998); *J. Theor. Biol.* **194**, 561 (1998); A. Lotem, M. A. Fishman, L. Stone, *Nature* **400**, 226 (1999); C. Wedekind and M. Milinsky, *Science* **288**, 850 (2000); A. Zahavi, A. Zahavi, A. Balaban, *The Handicap Principle* (Oxford Univ. Press, London, 1996).
22. T. Ellingsen, *Q. J. Econ.* **112**, 581 (1997); H. P. Young, *J. Econ. Theory* **59**, 145 (1993); A. Banerjee and J. Weibull, in *Learning and Rationality in Economics*, A. Kirman and M. Salmon, Eds. (Blackwell, Oxford, 1995); K. Binmore, A. Shaked, J. Sutton, *Am. Econ. Rev.* **75**, 1178 (1985).
23. From the huge literature on this topic, we quote two classics: T. C. Schelling, *The Strategy of Conflict* (Harvard Univ. Press, Cambridge, MA, 1960); R. Frank, *Passions within Reason* (Norton, New York, 1988).
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