Mathematics 113 Fall 2002

HOMEWORK ASSIGNMENT # 8
DUE, Thursday, November 21

**Collaboration**: On the homework sets, collaboration is not only allowed but encouraged. However, you must write up and understand your own individual homework solutions, and you may not share written solutions. If you learn how to solve a problem by talking to a classmate, CA, or looking it up in a book, you should cite the source in your homework write-up, just as you would reference your sources in a literature or history class.
Show all your working, and write up your solutions as neatly as possible.

1. Prove that
$$\sum_{n=2}^{\infty} \frac{1}{\binom{n}{2}^3} = \frac{1}{1^3} + \frac{1}{3^3} + \frac{1}{6^3} + \frac{1}{10^3} + \ldots = 8(10 - \pi^2).$$

2. Evaluate the integral
$$\int_0^\infty \frac{\log(1+x)}{1+x^2}.$$

3. Let $a > 0$ be real. Prove that
$$\int_0^\infty \frac{\sin ax}{e^x + 1} = \frac{1}{2a} - \frac{\pi}{2 \sinh \pi a}.$$

4. Prove that
$$\int_0^1 \log \Gamma(z) dz = \frac{1}{2} \log(2\pi).$$