Mathematics 113 Fall 2002

HOMEWORK ASSIGNMENT # 6
DUE, Thursday, November 7

Collaboration: On the homework sets, collaboration is not only allowed but encouraged. However, you must write up and understand your own individual homework solutions, and you may not share written solutions. If you learn how to solve a problem by talking to a classmate, CA, or looking it up in a book, you should cite the source in your homework write-up, just as you would reference your sources in a literature or history class.
Show all your working, and write up your solutions as neatly as possible.

1. Let $f(z)$ be a holomorphic function on the unit disc such that $|f(z)| = 1$ for all $|z| \leq 1$. Prove that $f(z)$ is constant, using:
   (a) The Open Mapping Theorem.
   (b) The Cauchy–Riemann equations, and the identity
   $$|f(x+iy)|^2 = u(x,y)^2 + v(x,y)^2.$$
2. (a) Let n be a positive integer. Evaluate the Integral
   $$\int_{-\infty}^{\infty} \frac{1}{x^{2n}+1}.$$
   (b) Given that
   $$\lim_{n\to\infty} \frac{1}{x^{2n}+1} = \begin{cases} 0, & |x| > 1, \\ 1, & |x| < 1. \end{cases}$$
   Find the limit of the integral from part (a) as n approaches infinity, and show that it agrees with your answer.
3. Let $f(z)$ be a holomorphic function such that $f(z) \neq 0$ for all z. Let $g(z) = \log f(z)$. Prove that $g(z)$ extends to a holomorphic function for all z.