Mathematics 113 Fall 2002

HOMEWORK ASSIGNMENT # 1
DUE, Thursday, September 26

Collaboration: On the homework sets, collaboration is not only allowed but encouraged. However, you must write up and understand your own individual homework solutions, and you may not share written solutions. If you learn how to solve a problem by talking to a classmate, CA, or looking it up in a book, you should cite the source in your homework write-up, just as you would reference your sources in a literature or history class.
Show all your working, and write up your solutions as neatly as possible.

1. Solve the following equations in complex numbers, and write the answer in polar form and rectangular form respectively.
   (a) $z^2 - i = 0$.
   (b) $z^7 + z^6 + z^5 + z^4 + z^3 + z^2 + z + 1 = 0$.
   (c) $e^{2z} + 2e^z = -2$. (Omit writing z in polar form for this question.)
   (d) $(z+1)/(z-1) = e^{\pi i/3}$.

2. Let $F(x) = a_0 + a_1 x + \ldots a_n x^n$ be a polynomial with real coefficients. Suppose that $z = a + bi$ is a root of $F(x)$. Prove that $\overline{z} = a - bi$ is also a root.

3. Let $z = a + bi$ with a and b real. What is the real part of $\cos(a + bi)$?

4. Let x and y be any two non-zero complex numbers. If we define $x^y := e^{y \cdot \log x}$, then x^y is not well defined due to the ambiguity in the logarithm. What are the possible values of i^i?

5. Let $\omega = e^{2\pi i/p}$ be a pth root of unity. Let
$$\chi(p) = \sum_{n=0}^{p-1} \omega^{n^2}.$$
Show that $\chi(3)^2 = -3$, $\chi(5)^2 = 5$ and $\chi(7)^2 = -7$. The expression $\chi(p)$ is known as a Gauss sum. For odd primes p, it turns out that
$$\chi(p) = \sqrt{(-1)^{\frac{p-1}{2}} p}.$$

6. Prove that
$$1 + \cos\theta + \cos 2\theta + \ldots + \cos n\theta = \frac{\sin\frac{1}{2}(n+1)\theta \cdot \cos\frac{1}{2}n\theta}{\sin\frac{1}{2}\theta}$$