Final, Mathematics 101

Name:________________________

January 8, Due: Friday, January 12

Problem 1.

Show that for any three nonempty sets X, Y, Z

$$\left|\left(X^Y\right)^Z\right| = \left|X^{(Y \times Z)}\right|.$$

Problem 2.

Show that any set of pairwise nonintersecting disks on the plane is countable.

Problem 3.

Construct a set of sets A such that $\cap A = \emptyset$, but for any countable subset $B \subset A$ $\cap B \neq \emptyset$.

Problem 4.

Show that any two cycles σ_1 and σ_2 of the same length in S_n are conjugate, i.e. there exists $g \in S_n$ such that $\sigma_1 = g\sigma_2 g^{-1}$.

Problem 5.

Show that if N and M are normal subgroups of a group G with $M \cap N = \{e\}$ then every two elements $m \in M,\ n \in N$ commute.

Problem 6.

Describe all cyclic subgroups of D_n.

Problem 7.

Prove that a circle is not homeomorphic to an annulus.