Homework #3

Due Thursday, October 12, in class
Problem 1 Let A be a set with n elements.

a) How many functions $f : A \to A$ are there?

b) How many of them are injective?

Problem 2 Let A be a set. What functions $f : A \to A$ are also equivalence relations?
Problem 3 Let $A, B,\ C$ be sets, $f : A \to B$ and $g : B \to C$ be functions.

a) Prove that if $g \circ f$ is injective then both f and g are injective.

b) Discuss the assertion "$g \circ f$ is surjective iff f and g are surjective".

c) Suppose $h : B \to C$ is a function and $h \circ f = g \circ f$. Show that if f is surjective then $g = h$.

Problem 4 Let A be a set. List all the elements of $A^{\emptyset}$, $\emptyset^{\emptyset}$, $\emptyset^{A}$.
Problem 5 Define $f : \mathbb{Z} \to \mathbb{N}$ by $f(x) = -2x$ for $x \leq 0$ and $f(x) = 2x - 1$ for $x > 0$. Prove that f is a bijection.
Problem 6 Describe the sets $\cup\mathbb{N}$ and $\cap\mathbb{N}$.
Problem 7 Which set has more elements: $\mathbb{N}^2$ or $2^{\mathbb{N}}$?