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Name:

LINEAR ALGEBRA AND VECTOR ANALYSIS

MATH 22B

Total:

# Unit 14: Hourly 1

## Problems

**Problem 14.1 (10 points):**
Prove by induction that that for every $n \geq 1$ the formula $2\sum_{k=0}^{n-1} 3^k = 3^n - 1$ holds.

**Problem 14.2 (10 points):**
a) (5 points) Row reduce the matrix $A = \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 4 & 4 & 4 & 4 & 4 \\ 2 & 2 & 2 & 2 & 2 \end{bmatrix}$ using basic row reduction steps.
b) (5 points) For $B = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$ compute either $AB$ or $BA$ depending on which of the two makes sense.

**Problem 14.3 (10 points):**
a) (2 points) Parametrize the curve $4x^2 + y^2 = 1$ in $\mathbb{R}^2$.
b) (2 points) Parametrize the curve $y - e^x = 0$ in $\mathbb{R}^2$.
c) (2 points) Parametrize the curve $x = y^3, z = 4$ in $\mathbb{R}^3$.
d) (2 points) Parametrize the line $x + y = 4, z = 2$ in $\mathbb{R}^3$.
e) (2 points) Parametrize the circle $x^2 + y^2 + z^2 = 4, z = 1$ in $\mathbb{R}^3$.

**Problem 14.4 (10 points):**
a) (8 points) Compute arc length of $r(t) = \begin{bmatrix} \frac{t^3}{3}, \sqrt{2}\frac{t^4}{4}, \frac{t^5}{5} \end{bmatrix}$ for $0 \leq t \leq 1$.
b) (2 points) Without doing any calculation, what is the arc length of the new parametrization $r(t^3)$ with $0 \leq t \leq 1$.

**Problem 14.5 (10 points):**
a) (2 points) Formulate the Al Khashi formula.
b) (2 points) We have seen a theorem of Heine- ....... . Fill in the second name!
c) (2 points) The linear space $\{x, Ax = 0\}$ is also called the ...... of $A$.
d) (2 points) Give the Euler's formula $e^{it} = \ldots\ldots$ and deduce the "most beautiful formula in math".
e) (2 points) Is $\text{rref}(A) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ row reduced?

**Problem 14.6 (10 points):**
a) (2 points) Express $z = e^{i\pi/2} + 3e^{i\pi}$ in the form $z = a + ib$.
b) (2 points) Write $(r, \theta, z) = (2, -\pi/2, 0)$ in Cartesian coordinates.
c) (2 points) Write $(x, y, z) = (2, 2, 0)$ in spherical coordinates $(\rho, \phi, \theta)$.
d) (2 points) Write the surface $\rho\cos(\phi) = 2$ in Cartesian coordinates.
e) (2 points) Write the surface $r\cos(\theta) = 2$ in Cartesian coordinates.

**Problem 14.7 (10 points):**
a) (5 points) You are given $r''(t) = \begin{bmatrix} 0 \\ 1 \\ \cos(t) \end{bmatrix}$ and $r(0) = \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}$ and $r'(0) = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$. Find $r(1)$.
b) (2 points) Is there a time $t$ such that the curve $r(t)$ ever reaches the ground $z = 0$?
c) (3 points) What is the curvature of $r(t) = \begin{bmatrix} t^2 \\ \cos(t) \\ \sin(t) \end{bmatrix}$ at $t = 0$?

**Problem 14.8 (10 points):**
We parametrize some surfaces. Chose the parameters on your own.
a) (2 points) Find a parametrization of the hyperboloid $x^2 + y^2 - z^2 = 1$.
b) (2 points) Find a parametrization of the cylinder $(x-1)^2/4 + y^2/9 = 1$.
c) (2 points) Find a parametrization of the surface $z = \cos(xy)$.
d) (2 points) Find a parametrization of the plane $x + y - 3z = 1$.
e) (2 points) Find a parametrization of the cylinder $x^2/9 + (y-2)^2 = 1$.

**Problem 14.9 (10 points):**
a) (4 points) Compute the dot product (inner product) $A \cdot B = \mathrm{tr}(A^T B)$ of the two matrices

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 & 2 \\ 2 & 3 & 3 \end{bmatrix}.$$

b) (4 points) Now determine the cosine of the angle between $A$ and $B$.
c) (2 points) Finally find the distance $|A - B|$ between $A$ and $B$.

**Problem 14.10 (10 points):**
a) (4 points) What is the Jacobian matrix $dr$ of the coordinate change

$$r(\begin{bmatrix} x \\ y \end{bmatrix}) = \begin{bmatrix} 4x + y \\ y^2 \end{bmatrix} ?$$

b) (2 points) Now find the first fundamental form $g = dr^T dr$.
c) (2 points) Compute the distortion factor $|\det(dr)|$.
d) (2 points) Check in this case that $|\det(dr)| = \sqrt{\det(g)}$.