QUIZ 1 – Solutions

Algebra — Math 122 — Fall 2002

1. T. The alternating group A_3 is abelian. (In fact $|A_3| = 3$ so $A_3 \cong \mathbb{Z}/3$.)

2. F. If N is a normal subgroup of G, then $gn = ng$ for all g in G and n in N. (You only know $gN = Ng$.)

3. F. When G is nonabelian, its center $Z(G)$ is not a normal subgroup. ($Z(G)$ is always normal.)

4. F. The group S_3 is isomorphic to $\mathbb{Z}/2 \times \mathbb{Z}/3$. (The group S_3 is nonabelian.)

5. F. The rational numbers $(\mathbb{Q}, +)$ form a cyclic group. (What would the generator be?)

6. F. If H is a subgroup of G and K is a subgroup of H, then $[G : H] = [G : K][H : K]$. (For example if $H = G$ this would say $[G : G] = 1 = [G : K][G : K]$, which is wrong if $K \neq G$.)

7. F. The intersection $6\mathbb{Z} \cap 15\mathbb{Z} \cap 10\mathbb{Z}$ is $60\mathbb{Z}$. (It's $30\mathbb{Z}$, since $30 = \text{lcm}(6, 15, 10)$.)

8. F. Let H be the subgroup $\{-1, 1\}$ in $G = \mathbb{R}^*$. Then G/H is isomorphic to G. (G/H is isomorphic to the positive reals, not all nonzero reals.)