Homework – Week 7
Sets, Maps and Knots
Math 101 – Harvard University

1. Show that if $\sigma = (a_1 a_2 \cdots a_k)$ is a cycle of length k, then σ is an even permutation if k is odd, and vice–versa.

2. (i) What is the maximum possible order of an element of S_7? Give an example of an element with this order.

   (ii) Give an example of two elements of S_7 with the same order, but different parities.

3. Prove that D_{2n} is generated by the elements f and $r^i f$ whenever $\gcd(i, n) = 1$.

4. (Review.) Define $f : \mathbb{Z}/n \to \mathbb{Z}/n$ by $f(x) = 2x$. For what values of n is f injective? When is it surjective?

5. (Review.) Let $a \in G$ be an element of a group. Prove that the map $f : G \to G$ given by $f(x) = ax$ is a bijection. What is its inverse?

6. (Review.) Two elements x, y in a group G are said to be *conjugate* if there is a $g \in G$ such that $gxg^{-1} = y$. Prove that conjugacy is an equivalence relation.

7. (Review.) Let A and B be finite sets, with $|A| = a < |B| = b$. How many injective maps $f : A \to B$ are there?

8. (Review.) Given $n > 1$, define $f : \mathbb{C} \to \mathbb{C}$ by $f(z) = z^n$. For what values of $w \in \mathbb{C}$ is $G = f^{-1}(w)$ a subgroup of $(\mathbb{C}^*, \cdot)$? Show that when G is a group, it is also cyclic, and find its order.