

### Homework 13

Math 101 — Sets, Groups and Knots

*Due in class on Thursday, 1 December 2022*

1. Show how to change the crossings in the knot diagram  $9_{34}$  so the result is a diagram for the unknot.
2. Prove that the writhe of a knot does not depend on its orientation.
3. Compute the writhe  $w(K)$ , the bracket polynomial  $\langle K \rangle$ , and the polynomial  $X(K)$  for the unknot with two twists, shown in Figure 1.
4. Let  $L$  be the oriented unlink with two crossings shown in Figure 1. Compute  $w(L)$ ,  $\langle L \rangle$  and  $X(L)$ .
5. Let  $L_n$  be the unlink with  $n$  components. What is  $X(L_n)$ ?
6. Compute  $\langle 3_1^+ \rangle$  for the positive trefoil (all crossings safe).
7. Compute  $w(K)$ , and  $\langle K \rangle$  and  $X(K)$  for  $K = 4_1$  the figure eight knot. (Hint: use previous calculations of  $\langle K \rangle$  for the trefoil knot and the Hopf link. You should find that  $X(K) = A^{-8} - A^{-4} + 1 - A^4 + A^8$ .)

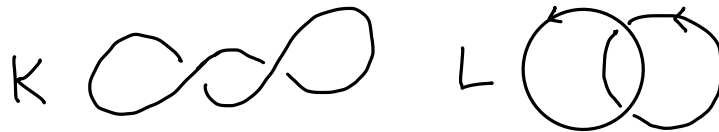


Figure 1. The unknot  $K$  and the unlink  $L$ , each drawn with two crossings.