Errata to the paper “Complete Kähler manifolds with nonpositive curvature of faster than quadratic decay”

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We would like to thank T. Ochiai for pointing out an error on p. 253. The inequality on p. 253, lines 7 and 8 from bottom, cannot remain valid when $\eta$ goes to $\infty$ with $\lambda$ fixed. To circumvent this difficulty, the proof of Proposition (2.4) on pp. 252-254 should be modified as follows:

We can assume (after choosing another point as $0$) that $r(x) = 1$ and $r(y) > 1$ for $y \in V$. For all $R > 0$, define

$$f_R(r) = c \int_r^R t^{-2n+1} \exp \left( A(2n - 1) \int_0^r \tau^{-2} u^2 (1 + u)^{-2} d\tau d\tau \right) dt,$$

where

$$c = \frac{1}{\omega} \exp \left( -A(2n - 1) \int_0^R \tau^{-2} u^2 (1 + u)^{-2} d\tau d\tau \right).$$

Then for some positive constant $E$ independent of $R$, we have

i) $f_R(R) = 0$,

ii) $\sup_{r \leq R} r^{2n-1} (1 + r)^{-i} f'_R(r) \leq E$,

iii) $-f'_R(r) \geq E^{-1} r^{-2n+1}$ for $r \leq R$,

iv) $(-f'_R(R)) R^{2n-1} \leq E$,

v) $\Delta f_R(r) + \delta_0$ is a nonnegative integrable function $g_R(r)$ on $B(0, R)$ satisfying $\sup_{r \leq R} g_R(r) r^{2n-1} (1 + r)^{i\tau} \leq E$, where $\delta_0$ is the Dirac delta function at 0.

These conditions are readily verified by direct computation. For the verification of v) one has to use

$$\Delta r = \frac{2n - 1}{r} + \frac{\partial}{\partial r} \log \sqrt{G_0}$$

and

$$f'_R(r) = -f''_R(r) \left( \frac{2n - 1}{r} + \frac{A(2n - 1)}{r^2} \int_0^r \frac{u^2 du}{(1 + u)^{2+i}} \right).$$

The motivation for $f_R$ is the transplantation to $M$ of the Green's function of a radially symmetric Riemannian manifold where the radial derivative...
of the logarithm of the volume element is less than or equal to
\( A(2n - 1)/r^4 \int_0^r u^2 du/(1 + u)^{2+\varepsilon} \). However, this background information is not needed.

By Green's formula and i), we have
\[
\frac{1}{2\pi} \int_{B(0, r)} \Delta f_R(r) \log |s| - \frac{1}{2\pi} \int_{\partial B(0, r)} f_R(r) \Delta \log |s| = \frac{1}{2\pi} \int_{\partial B(0, r)} f_R'(r) \log |s|.
\]

It follows from v) and the formula of Poincaré-Lelong that
\[
(\ast\ast) \quad \frac{1}{2\pi} \int_{B(0, r)} g_R(r) \log |s| - \frac{1}{2\pi} \log |s| (0) - \int_{\partial B(0, r)} f_R(r)
\]
\[+ \frac{\sqrt{-1}}{2\pi} \int_{B(0, r)} f_R(r)u = \frac{1}{2\pi} \int_{\partial B(0, r)} f_R'(r) \log |s|.
\]

By (1.10),
\[
\text{Vol}(V \cap B(0, r)) \geq \frac{\omega}{2\pi} (r - 1)^{2n - 2} \quad \text{for} \quad r \geq 1.
\]

Integrating by parts and using i) and iii), we obtain for \( R \geq 1 \),
\[
\int_{V \cap B(0, r)} f_R(r) = -\int_1^R f_R'(r) \text{Vol}(V \cap B(0, r)) \, dr
\]
\[\geq \frac{E^{-1} \omega}{2\pi} \int_1^R r^{-2n+1}(r - 1)^{2n - 2} \, dr
\]
\[\geq c_1 \log (1 + R),
\]
where \( c_1 \) is a positive number depending only on \( E \) and \( n \). By ii),
\[
\left| \int_{B(0, r)} f_R(r)u \right| \leq \omega D \Delta \int_0^R \frac{dr}{(1 + r)^{1+\varepsilon}} \leq \frac{\omega D \Delta}{\varepsilon}.
\]

By iv),
\[
-\frac{1}{2\pi} \int_{\partial B(0, r)} f_R'(r) \log |s| \leq \frac{1}{2\pi} (-f_R'(R)) \int_{\partial B(0, r)} \log^+ |s|
\]
\[\leq \frac{E \Delta}{2\pi} (\nu \log (1 + R) + \log C).
\]

By v),
\[
\frac{1}{2\pi} \int_{B(0, r)} g_R(r) \log |s| \leq \frac{E \Delta}{2\pi} (\nu \log (1 + R) + \log C) \int_0^R \frac{dr}{(1 + r)^{1+\varepsilon}}
\]
\[\leq \frac{E \Delta}{2\pi \varepsilon} (\nu \log (1 + R) + \log C).
\]

Putting these estimates into (\ast\ast) and letting \( R \to \infty \), we obtain a contradiction if
\[ \nu < 2\pi c \left( E \Lambda (1 + \varepsilon^{-1}) \right)^{-1}. \]

The following is a list of typographical errors unrelated to the error pointed out by Ochiai.

p. 226, line 16, \((1 - |z|^2)^*\) should read \((1 - |z|^2)^-\).

p. 240, line 10 from bottom, \(2^{2^{-1}}\) should read \(2^{2^{x^{-1}}}\).

p. 247, line 9 from bottom, \(|dz| \leq 1\) should read \(|dz| \leq 2\).

p. 252, line 10, \(\log s\) should read \(\log |s|\).

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