MATH IN THE TIME OF PLAGUE

BARRY MAZUR

In times like the present—with microscopic hedgehogs of coronavirus short circuiting our world; disintegrating our communities; threatening our families; casting a shadow on the present for our elders, on the future for our youth; immobilizing our economy; and poised to challenge the springs and gears of our democracy—our thoughts have certainly changed in mood, if not in substance.

Today's announced unemployment rate is a number I hesitate to write. It is proof enough that some of the less fortunate in our society are bearing the greater burden. One wonders whether this is a given in any plague; how many low-paid Egyptian laborers were collateral damage in the tenfold volley of altercation between God and Pharoah?

To defy all this, we should be looking out for what we can do for the good of others.... but also: we could be looking in, for some mode of consolation. Why don't we look about for gems of constancy—in our thoughts, expectations, in our ways of understanding the world—and rejoice in them—even the tiniest of them—as sparkles affirming the robustness of our souls?

Our daily missions have changed drastically:

Healers are at the front lines; as are any who provide equipment for them; as are many people in the applied sciences—all of these now feeling the urgent responsibility, and hence the strain, to produce, and to produce rapidly, helpful things to deal with this crisis.

Parents (and grandparents) have become homeschool teachers. Being bound in the nutshell of one's own house and yet exploring, in companionship with a First-Grader a world of mesozoic pterosauria (a world entirely unknown to me until a week ago) is thrilling, even though any metaphoric connection to the equally unknown future of this world had best be kept at bay.

Our ways of coming together have transformed:

The theaters, of course, are shut down. As for performances, well, there are fixed stage-blocking readings of Shakespeare plays, the actors in Zoom gallery view; e.g., *Two Gentlemen of Verona* with the by-line: *The Show Must Go On-Line* (in Zoom[1]). This follows good tradition, in that Shakespeare himself seemed to adapt well to the quarantine of plague after plague[2].

Thankfully, there still is, there always is, music: the food—yes, of love—but also, perhaps now, of comfort—perhaps of nostalgia. There are Yo-yo Ma's wonderful *Songs of Comfort in global crisis*. Or Neopolitan neighborhoods singing in harmony, yet separated by their balconies[3]

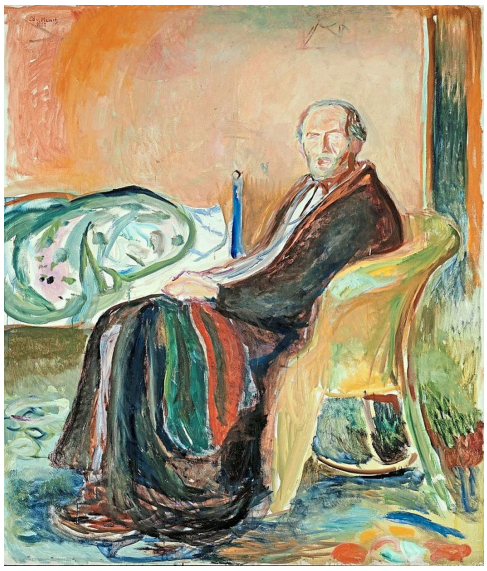
There will surely be Art (of the traditional sort, and—very likely—of novel forms as well) emerging from, and recording, our predicament,

[1] https://www.timeout.com/london/news/a-live-streaming-reading-group-is-performing-all-of-shakespeares-plays-in-order-031920

[2] in life, and in metaphor; see Stephen Greenblatt's striking *What Shakespeare Actually Wrote About The Plague* (The New Yorker; May 7, 2020) https://www.newyorker.com/culture/cultural-comment/what-shakespeare-actually-wrote-about-the-plague

[3] or this serenade of *Nessun Dorma* to hospital workers in the courtyard of a Warsaw hospital: https://www.thefirstnews.com/article/tenor-moves-frontline-medical-workers-to-tears-with-powerful-nessun-dorma-serenade-12923.

as there has been in earlier plagues. Here [4] is Edvard Munch's 'Self-Portrait After The Spanish Flu' (1919):



Our reception of any imaginative offering—literature, music, art—is molded by our experiences intermingled with our own imagination. Unsurprising that a pandemic might impel us to have quite a changed tone of appreciation of them. And even objects that have the stubborn staying power as mute inhabitants of the material world—even they—change, when viewed in a time of plague. Proust comments in his great novel: "Certain people, whose minds are prone to mystery believe that objects retain something of the eyes which have looked at them." I suppose that our gazes in the midst of this pandemic can transform—before our eyes—even the most robust object.

Maybe our sense of community; maybe our idea of justice—and how the mechanism of justice should be protected, and more radically: what

[4]This is a close-up view of that painting taken from an article *Five great painters who did art in times of plague* by Rouwen Lin in the online journal *The Star*: `https://www.thestar.com.my/lifestyle/culture/2020/03/24/five-great-painters-who-did-art-in-times-of-plague`.

See also: *Drawing Lessons in Time of Plague*: `https://www.google.com/search?q=drawing+lessons+in+times+of+plague&rlz=1C5CHFA_enUS517US517&oq=Draw&aqs=chrome.1.69i57j69i59l2j0j46j0l2j69i60.3730j0j7&sourceid=chrome&ie=UTF-8`.

all that boils down to meaning—maybe all that will change. Our recognition of the role of government in our common vulnerability is already changing.

Panta rhei—all things change.

Well, not quite.

I'm guessing—hoping—that any of our interests, and no matter what it is that we devote ourselves to, have core sensibilities—untouchable by external ravages. Those gems of constancy. It could be an interesting exercise, then, to examine what they are—to appreciate how valuable they are to our thoughts, and to our being.

Adam Smith, in *The Wealth of Nations* picks out what he calls a *propensity in human nature:* "the propensity to truck, barter, and exchange one thing for another," but then Smith steps back from affirming it as "one of those original principles in human nature."

Seeking more firmly rooted propensities in human nature, isn't there an imperturbable essence, for example in our impulse to storytelling? As Boccaccio proclaims in the 'Proem" that launches his hundred tales:[5]—this impulse has

> altogether or in part, power to draw the mind unto itself and to divert it from troublous thought, at least for some space of time, whereafter, one way or another, either solacement superveneth or else the annoy groweth less.

And even behind this, there is a primal mimetic urge—the germ of any artform according to Aristotle. This is an urge we all have—in some form, even if only exercised in our imagination; recollection itself being the most primitive mimetic act. The manner of expressing our common instinct to re-create and project images or experiences, or emotions is as varied as human experience, but the very kernel instinct—of mimesis, or re-presentation—remains "an ever-fixed mark" and unwavering in all of us.

[5] *The Decameron of Giovanni Boccaccio.* The quotation comes from the 1886 translation by John Payne: `https://www.gutenberg.org/files/23700/23700-h/23700-h.htm#Proem`

And then, mathematical thought. How peculiarly steadfast mathematical concepts are—let alone its truths.

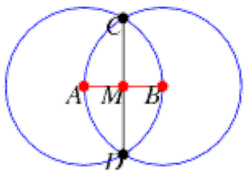
Of course, as with everything we humans do, the way we express our mathematics might evolve.

The vocabulary, the setting, the very attitude toward those activities and people's emotional response to them may vary from generation to generation. Cardano (16th c.) exhorted his readers to *dismiss mental tortures* so that they could bear to use the square root of negative 15 in a specific computation—whereas already two or so centuries later, mathematicians, physicists and engineers would greet complex numbers with delight, rather than anguish.

But what about our primal sense of those underlying concepts? No matter how you frame any movement-of-thought in mathematics—for example: constructions as in Euclidean Geometry, or in Algebra, or in any mathematical theory— its underlying meaning seems impervious to worldly vicissitudes.

This holds as well for the most elementary constructions; think of the activity of considering *twice* something—whether you label it *zweimal* or *deux fois*, or give it no particular label—the fundamental act of conceiving of the *double* of a quantity—be it a *number* (formulated and denoted however you want to formulate or denote numbers) or be it a *geometric entity*—has an unswerving intent, and meaning. This process of doubling, even though you might dismiss it as extremely simple, is an excellent example of a mathematical step-of-thought, and it shares—at least for the purpose of this discussion—all the essential qualities of any of the less readily graspable acts of the mathematical imagination.

Or, going the other way, consider the process of halving. Euclid begins his journey through the elements by finding "half" of a line segment by performing two swipes of a compass creating a perpendicular bisector—(despite the fact that nowhere in his axiomatic set-up is there any hint that two circles can intersect):



Simple constructions even simpler than these—or others that we hardly catalogue as constructions—without even depending on any clear prior structure—lodge in our imagination; they form, in one way or another, (what I want to call) our common mathematical sensibility. Admittedly, this sensibility is more developed in certain people than in others but I think there is an unperturbable quality to it—and we all have some of it.

As we navigate the world we can't help putting order (and finding order: patterns, rhythms, symmetries) in our thoughts about the objects, or ideas, that we encounter. That's not yet mathematics. It only becomes so when we make that tiny leap of level, and dwell on the "order or patten or rhythm or symmetry' as a thing-in-itself separate from the object or idea for which it was, initially in our thoughts, only a property.

The examples I just gave (*doubling* and *halving*)—these extremely primitive bedrocks of thought—lie before any of the grand constructions of the subject. And earlier than the very idea of proof.

We're certainly lucky to have rigorous mathematical proof. It models, and captures the essential quality of any reasoned argument that is careful to make transparent the prior stipulated truths on which that argument rests. How magical it is that the sharp edge of mathematical proof very often achieves universal agreement. But

- setting aside the great mathematical analogies that link different sources of intuition such as geometry and algebra,
- setting aside the various attitudes towards the nature of mathematics, attitudes that carry the labels platonism, or intuitionism,

- setting aside the history of self-entanglement issues in the foundations of mathematics,
- and even ignoring, for the moment, the weight Kant puts on his grand opening question in the *Critique of Pure Reason*, "How is Pure Mathematics Possible," thereby subtly stipulating—that Pure Mathematics is, in fact, Possible[6]—

there is, I think, a basic mathematical sensibility residing in all of us—expressed by the many (often unnoticed) expressions of our mathematical instincts. Such instincts, almost not describable in language, may well be the grounding of what we value in rational thought: instincts (pronoetic, as my son Zeke used to say) that seem to be rooted in times even prior to the emergence of intellect itself. At the very least, this capability, this sensibility, hasn't changed—I'm sure—from the epochs even before the days when the Pythagoreans were investigating triangles. There's a universal firmness to even the most elementary mathematical thought that makes it seem to be untouched by Time. And by Plague. Mathematics is"Omnitemporal," to use a latinate word which is one way of translating Edmund Husserl's more gemütlich "Allzeitlich,"[7].

In our current distancing social existence, with schools closed down, and where Zoom classrooms sometimes have the feel of Town Hall meetings... and sometimes the feeling of Quaker meetings, these new 'feels' of classrooms are modifying how we teach–what it means to teach. There are many wonderful ways to teach; and: to learn.[8]

Very close friends of mine, Bob and Ellen Kaplan, have run classes for children, with ages ranging from 4 to 14, called *Math Circles*[9]. The Kaplans have held these classes for decades, but had them converted to Zoom even prior to the appearance of Covid-19. The manner that

[6]and the consequences he draws from this.

[7]which is itself resonant with St. Augustine's discussion in the *Confessions* about the perplexity one has in understanding God's relation to time.

[8]If I had had a chance to nudge Meno to sharpen the question that kickstarts the platonic dialogue that carries his name, I'd have suggested he ask: "How can I *learn* virtue?"

[9]their math circles have now propagated all over the world: `https://www.globalmathcircle.org`

Bob would start a class of 4 and 5 year old children is to stride into the classroom—i.e., now the screen—and confidently write in large print:

$$1 \quad 2 \quad 3 \quad 5 \quad 6 \quad 7 \quad ---$$

By the time the number 7 was inscribed, some child would shout "but you forgot 4." Bob would strike his forehead with his hand and proclaim "Oh yes: you're right. Of course there's a number between 3 and 5" as he inserted a "4" in the appropriate spot. And without losing a second would add "but there's surely no number between 4 and 5." Some child would invariably counter that by offering "I'm four and a half," at which point Bob's hand would go to his forehead again, as he corrected that second mistake, further revising his increasingly crowded number line. It is in this elegant via negativa type approach that by a seemingly never-ending series of Bob's mistakes corrected by the children, a great many mathematical concepts would be constructed, examined, and would lead to further questions.

What power of illumination error possesses! Learning from error is one of the many daily delights we all have. Mathematicians, of course, too—what with indirect argument as one systematized embodiment of math "learning from error."

Sometimes, in trying to show that a certain something—endowed with specific features—doesn't exist, mathematicians, having lived—perhaps for years— with such a nonexistent object, and having established various properties that it would have if it existed, give it a congenial name—even if it has no "local habitation." Such (conjecturally nonexistent) 'yet familiar' objects become pets—so to speak—destined to be shown to be will-o'-the-wisps.

For example, analytic number theorists talk about something they call *the ghost zero* which if it existed, would carry consequences that we should be aware of; it surely doesn't exist—but we don't know that yet. I would think that all mathematicians have encountered and dwelt with some such object—or perhaps a number of them—having specific properties (if they existed) but where the important mission is to show nonexistence; that they are *ignes fatui.* [I lived day-and-night for a number years with at least one such creature that was—decades later—happily and finally shown to *not exist* by Perelman's proof of the three-dimensional Poincaré Conjecture.]

How can mathematicians be of help in our current pandemic? Here—as always—there is a crucial distinction between pure and applied mathematics, the latter being of critical importance, a treasure trove of—to use Benjamin Franklin's cherished humble-seeming phrase—*useful knowledge*.

Applied mathematicians have been pressed into service; how grateful we all are for the precision of immunological and epidemiological studies—the collecting, sorting, classifying, and interpreting data. And for the formulation, and calibration, of models that help in interpreting what the data wants to tell us about what has happened in the past and what we can expect for the future.

So then, how can we pure mathematicians be of help? Besides, of course, teaching Multivariable Calculus and Probability Theory to the future generation of epidemiologists and practitioners, and just homeschooling their children or grandchildren and keeping contact with all their students; usually necessarily Zoom contact.

As for this new Zoom epoch, there are many in our mathematical community—including undergraduate math majors[10], graduate students and professors—who are taking the initiative[11] to make use of this moment when math classes are zooming all over the world to design fundamental online techniques to accommodate this moment, where standard courses may have international range—as does Ravi Vakil's (current online summer) algebraic geometry course (which has over 1600 signups around the world).

But theoretical mathematicians also can just try to be close and engaged listeners of the reports coming from their colleagues who are at work in applied directions. I personally intend to learn a bit about the mechanisms of forecasting during this pandemic to make up for my total lack of knowledge or experience about the handling of data in any form (and for any purpose).

Extremely good expositions are available, that assume absolutely nothing at all in the way of background knowledge, and rather rapidly

[10]See the Harvard Crimson article by Leo Alcock, a sophomore math major at Harvard: `https://www.thecrimson.com/article/2020/7/1/alcock-first-lockdown/`

[11]See the *Open Online Education Project* (OOEP) `ooep.org`; its mission is to expand and improve online education and to extend the breadth and depth of *free* course resources worldwide. This project was inspired by MIT's decades old *OpenCourseWare* `https://ocw.mit.edu/index.htm`.

get you (even if you are an extreme outsider) to be able to—and to want to—-dig into relevant data; for example: data regarding the evolution of particular strains of Covid-19.

One such exposition[12] is a You-Tube presentation (`https://www.youtube.com/watch?v=pg0wkFRBRt8`) by Pleuni Pennings (a researcher in San Francisco State University who studies the evolution of viruses—but not Covid-19). In this presentation she shows how to work with genetic evolutionary trees of Covid-19 developed by Trevor Bedford[13] and the team at `@seattleflustudy` who sequenced the genome of Covid-19. She explains how to use that online database to trace a mutation at a point of the genetic code of the virus as it travels its way through Washington State, and trace it back to antecedents in China.

When it comes to reading about models and forecasts based on models, I feel that I, as an outsider, need more guidance to understand them than is often available.

For example, in forecasts that give, say, an estimate (within 95% expectation) of the range of uncertainty[14], then (if both the model used for forecasting and the circumstances regarding the data collected haven't changed in any significant way in the recent past) it might be useful to the nonprofessional readers like me if easy access to the past forecasts were also offered—i.e., answering the question: how well, in the forecasts given in days or weeks past, did the model's projected range of uncertainty in those forecasts encompass the actual data.

And contemporary models are pretty complex—not as elementary as the first mathematical models in the modern era, like the one outlined in Thomas Malthus's (*An Essay on the Principle of Population 1798*). Malthus's model has two transparent variables, and none hidden. These are governed by his two starting postulates:

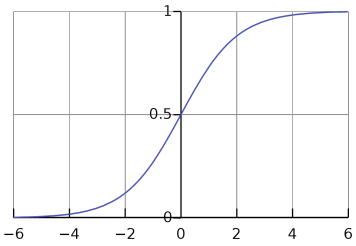
- First, that food is necessary to the existence of man.
- Secondly, that the passion between the sexes is necessary and will remain nearly in its present state.

[12]mentioned to me by the statistician Susan Holmes.

[13]Trevor Bedford is an evolutionary biologist Virus tracking expert. To work with his team's evolutionary tree, click on `https://nextstrain.org/ncov/global`.

[14]e.g., as in `https://covid19.healthdata.org/united-states-of-america`

The second postulate would have population increasing exponentially, while the first is what tempers that exponential growth, leading to the proliferation of sigmoid curves that dominate the subject—in today's parlance, it is the agent that "flattens the curve."



"Population, when unchecked," writes Malthus,

> increases in a geometrical ratio. Subsistence increases only in an arithmetical ratio. A slight acquaintance with numbers will shew the immensity of the first power in comparison of the second.

Modern research is nothing as simple as that; it involves astute manipulation of the models; curve-fitting—using the various parameters allowed by a host of 'hidden variables'; combining—with an experienced eye—data (of different sample sizes) from different sources (with different variances, and margins of error)[15].

As for "different variances," take the simple parameter labeled "R_0" the *basic reproductive number* of the virus, characterizing the average number of secondary cases generated by each primary case. (So, if R_0 were equal to 2 that would mean that, on the average, each of us, when infected, infects two others; this would be bad news even though it couldn't go on that way forever.)

[15]Today, for example, NPR reported on the work of Nicholas Reich, a biostatistician at University of Massachusetts Amherst, who with his colleagues have developed a method to merge the diverse models of the disease's progression into one "ensemble" projection.

> It's a sort of portal through which the scientists behind each COVID-19 model can communicate key details about their methodology and results, so that, as Reich explains, "all of these forecasts can be represented in a single standardized way. And this makes it really easy to make apples-to-apples comparisons between these models."

This single number R_0 is more informative—or less so—depending on how homogeneous the population is. If we average over conglomerates of different populations of different age-levels, with different levels of susceptibility, and—more to the point—different practices that produce different levels of exposure to individuals to get a single 'average number' R_0, it would be far less useful, than if one devised a hierarchy of separate analyses each focused on a different segment of the population these sub-populations representing more internal homogeneity—and perhaps significantly smaller variance in the data. One would end up with a hierarchy of R_0s that would convey real information, a worthy guide for possible action, or prediction—more informative than just a single overall average number. This is suggested in the paper *Modeling the Heterogeneity in COVID-19s Reproductive Number and Its Impact on Predictive Scenarios* of Claire Donnat and Susan Holmes `https://arxiv.org/pdf/2004.05272.pdf`[16].

Whether or not we understand models, they are all around us. Models organize our life via machine-learned stratagems; they tell us how to proceed in one activity or another. Some of these models trigger self-reinforcing feedback loops that tell us to do more of a certain activity if, simply, we've already done lots of it. A troubling example of this is in a predictive policing software "PredPol,"[17] that even according to the company that produced it, is based on seismic earthquake prediction, and—according to a statistician (Kristian Lum) who has looked at it—is "just a moving average" (i.e., takes an average of where arrests have already happened, and tells police to go back there). But—in the words of the company that produced it—it is:

[16]They deal with 19 geographical groups to gauge the amount of variability shown in the reproductive number:

- the six countries reporting the highest numbers for the epidemic in Europe (Italy, Spain, France, Germany, the United Kingdom and Switzerland),
- seven groups in Asia (Hong Kong, the Chinese provinces of Guizhou and Hubei, Singapore, Thailand as well as Japan and South Korea),
- Iran,
- the United States as a whole, as well as the states of California, Washington and New York.

[17]See: *Academics Confirm Major Predictive Policing Algorithm is Fundamentally Flawed* by Caroline Haskins Motherboard (2019) `https://www.vice.com/en_us/article/xwbag4/academics-confirm-major-predictive-policing-algorithm-is-fundamentally-flawed`

> based on nearly seven years of detailed academic research into the causes of crime pattern formation the mathematics looks complicated—and it is complicated for normal mortal humans—but the behaviors upon which the math is based are very understandable.

This, alone, is troubling talk, vaunting a level of incomprehensibility of the mathematics of their model—a model that encourages police to look for petty crime that may go unnoticed in other neighborhoods.

Happily, there are movements pressing for real reform: to have the police fulfill their function primarily as guardians rather than warriors[18]. And, perhaps inspired by this time of plague, even more: by protests resounding internationally, we are in the midst of a call for fundamental re-evaluation of the way we treat, or—to put it one step removed—"society treats" segments of humanity.

Times of plague are times of all kinds of re-evaluation. There is the legend that during the cholera epidemic of 1848 Rabbi Israel Salanter—who was the religious and ethical leader of the Jewish community in Vilna, at that time—publicly called for his community to eat, and *not to fast*, on the solemn fast day of Yom Kippur. This was so his community would not be made more vulnerable to the plague by a day of fasting[19]. The pinnacle of holy observance stands back, for the plague.

We have no idea how things will evolve, how we ourselves will be altered. In such a time, we can at least be thankful for the shared ideas that ground us, ideas that unite us; and for the unwavering solidity of certain keystones of thought[20]. And we can try to work—in whatever

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[18]`https://www.policeone.com/research/articles/is-there-any-evidence-concerning-the-warriorguardian-debate-in-policing-y9hPZYjiBHXrY0cB/`

[19]The primary school I went to was Yeshiva Israel Salanter in the Bronx—where Rabbi Salanter's humane *mussar* sentiments pervaded—in the midst of precise, demanding and all-encompassing religious ritual. Salanter's mantra—along with the standard 'love thy neighbor'—was the equally exacting"my neighbor's material needs are my own spiritual needs." My schooldays in *Salanter* had a transcendental electricity that I'm very thankful for... and continue to be puzzled by.

[20](even the simplest mathematical gems; e.g., the *Tetrahedron*, *Cube*, *Octahedron*, *Dodecahedron*, and *Icosahedron* that Euclid encountered at the end of his tour of the Elements)

way we can—towards a healthier (and more just!) global community—looking for the day when this flurry of pandemic forecasts we currently face become hindcasts; and this time of plague a memory.

June 30, 2020, Cambridge, Mass.