Lecture 22: Surface area

For a parametric surface, the surface area is defined as
\[ \int \int_R |\vec{r}_u \times \vec{r}_v| \cdot dudv . \]

For a surface of revolution parameterized by
\[ \vec{r}(\theta, z) = [g(z) \cos(\theta), g(z) \sin(\theta)] . \]
we get
\[ |\vec{r}_\theta \times \vec{r}_z| = |g(z)| \sqrt{1 + g'(z)^2} . \]

The surface area of such a surface of revolution is
\[ 2\pi \int_a^b |g(z)| \sqrt{1 + g'(z)^2} \; dz . \]

1. Find the surface area of the surface
   \[ \vec{r}'(u, v) = [u, v, 2u] \]
   with \(0 \leq u \leq 1\) and \(0 \leq v \leq 3\).

2. Find the surface area of the surface
   \[ \vec{r}'(u, v) = [u, v, u^2] \]
   where \(0 \leq u \leq 1\) and \(0 \leq v \leq u\).

Gabriel’s trumpet is the surface of revolution where \(g(z) = 1/z\), where \(1 \leq z < \infty\).

3. Verify that the volume of the trumpet is \(\int_1^\infty \pi g(z)^2 \; dz = \pi\).
Compute the surface area integral of the trumpet.

We conclude that the trumpet is a surface of finite volume but with infinite surface area! You can fill the trumpet with a finite amount of paint, but this paint does not suffice to cover the surface of the trumpet!