Unit 9: Parametrized surfaces

In order to study parametrized surfaces, it is helpful to look at grid curves. These are the curves obtained if one parameter is kept fixed. For a sphere, the circles of latitudes and half circles of longitudes are the grid curves. The equator is the circle of latitude of 0 which corresponds to $\phi = \pi/2$. The conversion is $\text{latitude} = \pi/2 - \phi$. The sphere is parametrized by

$$\vec{r}'(u, v) = [\cos(u) \sin(v), \sin(u) \sin(v), \cos(v)] .$$

We analyze now the surface

$$\vec{r}(u, v) = [u \cos(v), u \sin(v), v] .$$

1. What are the grid curves where $u$ is constant, like $u = 1$?

2. What are the grid curves where $v$ is constant, like $v = 0$?

3. Can you draw the surface?

4. Let us change the parametrization to

$$\vec{r}(u, v) = [u \cos(v), u \sin(v), u] .$$

What surface is this now?